Fourier Integrals for Practical Applications

By

The Late GEORGE A. CAMPBELL

AND

RONALD M. FOSTER

Brooklyn Polytechnic Institute

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Fourier Integrals for Practical Applications

ABSTRACT: The growing practical importance of transients and other nonperiodic phenomena makes it desirable to simplify the application of the Fourier integral in particular problems of this kind and to extend the range of problems which can be solved by this method in closed form. To facilitate the use of the known closed form evaluations of Fourier integrals many of them have been compiled in Table I. Special attention has been given to including every limitation and every warning which may be necessary for the safe use of each integral. This required a rigorous checking of the evaluations of the integrals. A few Fourier series have been included in Table I and also certain contour and indefinite integrals. Applications of Fourier integrals to 85 transient problems are given in Table II.

INTRODUCTION *

THE Fourier integral and the Fourier series are alternative expressions of the Fourier theorem, the series being a limiting case of the integral and vice versa. Usually the theorem is approached from the side of the series, but there are also advantages in the approach from the integral side, which is the method followed in this paper. The generality and importance of the theorem is well expressed by Kelvin and Tait who said: ". . . Fourier's Theorem, which is not only one of the most beautiful results of modern analysis, but may be said to furnish an indispensable instrument in the treatment of nearly every recondite question in modern physics. To mention only sonorous vibrations, the propagation of electric signals along a telegraph wire, and the conduction of heat by the earth's crust, as subjects in their generality intractable without it, is to give but a feeble idea of its importance." For any real understanding of the theorem it is necessary to appreciate why it is one of the most beautiful mathematical results and why it furnishes an indispensable instrument in physics.

The Fourier integral is a most beautiful mathematical result because

* A paper entitled "The Practical Application of the Fourier Integral" was prepared by us and presented by George A. Campbell, September 13, 1927, at the International Congress of Telegraphy and Telephony in Commemoration of Volta. After revision this was published in the *Bell System Technical Journal*, October, 1928, pages 639–707. Both editions of this earlier paper are completely replaced by the present paper, in which has been incorporated practically all of the material in them, including the 183 pairs (integrals) in Parts 1–9 of Table I which bear a number having no decimal part. This table now contains a total of 763 pairs.

of the economy of means employed in obtaining a most general result One form of integral is used both to analyze and to synthesize In both cases it is the product of the arbitrary function and the elementary sinusoidal oscillation which is integrated This achieves the mathematical counterpart of spectrum analysis and spectrum synthesis The functions resulting from analysis and synthesis stand in a mutually reciprocal relation[1] They are paired with each other Either of these functions may be assigned with an astonishing degree of arbitrariness Singular cases being excepted the mate function is then determined uniquely and definitely by the integral While the sine cosine and complex exponential are most commonly used as the elementary expansion functions an entire class of functions presents the same fundamental relations and finds applications in the more recondite problems

The Fourier integral is an indispensable instrument in connection with physical systems in which cause and effect are linearly related (so that the principle of superposition holds) because it gives at once an explicit formal solution of general problems in terms of the solution for the sinusoidal case which is often readily found This explicit general solution makes use of two Fourier integrals one for the spectrum analysis of the arbitrary cause and the other for the spectrum synthesis of the component sinusoidal solutions No further consideration of the actual physical system is necessary after the elementary sinusoidal solution has been obtained This point of view has become a part of our general background of thought

Unfortunately the actual evaluation of specific Fourier integrals in closed form presents formidable if not insuperable difficulties Only a small number of distinct general integrals have been evaluated in closed form in the century and more which has elapsed since the Fourier integral discovery was announced Additions to the list of evaluated Fourier integrals can ordinarily be made only by the professional mathematician Unless the physicist or technician is in a position to evaluate Fourier integrals by mechanical means or is satisfied to employ infinite series or other infinite processes in place of the definite integrals he is usually entirely dependent upon the evaluations which the professional mathematician has made in the past or is able to make for his special use On this account it is often desirable to so formulate practical problems that only evaluated Fourier integrals will occur It would be well for the physicist and technician to become

[1] The fundamental importance of the Fourier integral may be associated with an analogy which exists between the integral and the imaginary unit both considered as operators. In both cases two iterations of the operation merely change a sign and four iterations completely restore the original function

familiar with the Fourier integral evaluations which the professional mathematician has achieved.

It is the purpose of this paper to take the first steps towards the preparation of two tables, one giving the evaluations of Fourier integrals and the other giving the sinusoidal solutions for physical systems. Together they would reduce the practical application of the Fourier integral to the selection of three results from these two tables. Thus by means of the first table the arbitrary cause could be resolved into a sum of sinusoidal causes; by means of the second table the solutions for these sinusoidal causes could be supplied; and, finally, by means of the first table again, the effect of superposing these sinusoidal solutions could be shown, and thus the answer to the original problem would be given.

The preparation of the tables calls primarily for a compilation of the results already obtained by pure analysis, after which new evaluations and new solutions should be added, in so far as is possible. No attempt has yet been made to completely cover the existing literature on the subject, which extends back over one hundred years and is extensive and widely scattered. But sufficient has been done to show that the forms of the tables which are proposed are most convenient for practical application.

Paired Coefficients—Terminology

The Fourier integral theorem has been expressed in several slightly different forms to better adapt it for particular applications. It has been recognized, almost from the start, however, that the form [2] which best combines mathematical simplicity and complete generality makes use of the exponential oscillating function $e^{i2\pi ft}$. More recently the overwhelming advantage of using this oscillating function in the discussion of sinusoidal oscillatory systems has been generally recognized. It is, therefore, plain that this oscillating function should be adopted as the basic oscillation for both of the proposed tables. A name for this oscillation, associating it with sines and cosines, rather

[2] The form of the Fourier integral theorem referred to is: Subject to certain general restrictions, any function $G(t)$ can be expressed as the double integral,

$$G(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i2\pi f(t-g)}G(g)dg\,df.$$

For tabulation purposes in Parts 1–9 of Table I this double integral is broken up into two simple integrals which are written at the head of Table I in pair (101) and pair (102) as follows:

$$G(g) = \int_{-\infty}^{\infty} F(f)\ \text{cis}\ (2\pi fg)df,$$

$$F(f) = \int_{-\infty}^{\infty} G(g)\ \text{cis}\ (-2\pi fg)dg.$$

than with the real exponential function, seems desirable. The abbreviation cis x for $(\cos x + i \sin x)$ suggests that we name this function a cisoidal oscillation. This term is tentatively employed in this paper. The notation $\mathrm{cis}(2\pi ft)$ is also employed where it is desired to use an expression which is essentially one valued, which avoids the use of exponentials, or which suggests periodic oscillations by its connection with cosine and sine.[3]

In a table of Fourier integrals, every integral expression would then contain, in addition to the arbitrary function $F(f)$, the same oscillating function $\mathrm{cis}(2\pi ft)$, the same integral sign with limits $-\infty$, $+\infty$ and the same differential df. To repeat any such group of a dozen characters in each of several hundred entries seems quite unnecessary. It is, therefore, proposed merely to tabulate the arbitrary function $F(f)$ and the value $G(t)$ for the evaluated integral expressed as a function of the time. The table is thereby reduced to two parallel columns of associated functions, one of which is employed as the coefficient of the elementary cisoidal function while the other is a function of the independent time variable. The table would, however, be more symmetrical if both of the associated functions could be regarded as coefficients of an elementary function. This may be done by introducing the unit impulse as an elementary function, the impulse occurring at the epoch g at which instant it presents a unit area whereas its value is zero for all time before and all time after the epoch g. This is an essentially singular function and to recognize this fact it will be designated by $\mathfrak{S}_1(t-g)$ which is intended to emphasize the singularity. The time function may now be replaced in the table by the same function of the parameter g, since the time function $G(t)$ is equal to the integral with respect to g between infinite limits of the product $G(g)\mathfrak{S}_1(t-g)$.

The table of Fourier integrals has now become also a table of paired coefficient functions. This means that if the coefficient $F(f)$ is employed with the cisoid and the coefficient $G(g)$ is employed with the unit impulse and both products are summed for the entire infinite range of their parameters f and g, the same identical resulting time function is obtained.[4] Taken in connection with their respective ele

[3] Since the cisoidal oscillation is simply a uniform rotation at unit distance about the origin in the complex plane, it may be desirable to try some compact notation which directly suggests this rotation, for example ru(ft) 1^{ft}, i^{4ft} might be defined as the complex quantity obtained by rotating unity through ft complete turns or $4ft$ quadrants.

[4] The use of frequency and epoch as the two parametric variables gives us many symmetrical formulas where, if the radian frequency were employed, an unsymmetrical 2π would occur. In practical applications the frequency of the coefficient pairs becomes the frequency which is ordinarily employed in acoustics, in music and in commercial alternating currents. The basic unit for frequency is the reciprocal second, the unit for epoch is the second.

mentary functions, the two associated coefficient functions are, therefore, equivalent, alternative ways of representing a particular time function. This geometrical or physical point of view makes it natural to work directly with coefficient pairs without introducing the overworked integral sign in practical applications. For this reason, although the table has been headed a table of Fourier Integrals, it may equally well be considered to be a table of Paired Coefficients.

There is another fundamental reason for placing both of the functions $F(f)$ and $G(g)$ on the same footing as coefficients. It is this: Fourier's fundamental discovery was that the two functions may be transposed in the Fourier integral if the sign of one of the parameters is reversed. Thus, either one of the two functions constituting any coefficient pair may be taken as the coefficient of the cisoidal oscillation, provided only that the proper sign is given the epoch parameter occurring in the other function. For this reason also both functions are thus quite properly regarded as coefficients.

It is found convenient to call each coefficient of a coefficient pair the mate of the other coefficient, pair and mate being employed just as in the case of gloves. To find the mate of a glove, it is necessary to know all about the given glove including the fact as to whether it is the right or the left one of the pair. In the same way, to find the mate of a coefficient function, it is necessary to know not only the form of the function, but, in addition, whether its variable is the frequency or the epoch. The notation $\mathfrak{M}G(g)$, $\mathfrak{M}F(f)$ will be employed to indicate the mate of the particular coefficient $G(g)$, $F(f)$.

We have now defined and explained the proposed terminology for use in the practical application of the Fourier integral theorem. Before proceeding to practical applications, it is desirable to become familiar with these coefficient pairs considered in their own right. We may well begin by reminding the reader of the dissimilarity between the elementary oscillations.

The Two Elementary Functions Contrasted

The dissimilarity between the two elementary functions of the time, the cisoidal oscillation $\operatorname{cis}(2\pi ft)$ and the unit impulse $\mathfrak{S}_0(t-g)$ is most striking. This is clearly shown by the wire models of Fig. 1 where each function is depicted for five values of its parameter. For the value zero the cisoidal oscillation degenerates into an infinite straight line parallel to the time axis and cutting the real axis at $x = 1$. For the same value zero of its parameter g, the unit impulse is zero everywhere except at the origin where it has a vanishingly narrow loop extending to $x = +\infty$.

For other values of the parameter, the cisoidal oscillation is always an infinite cylindrical helix, centered on the time axis, and passing through the point $x = 1$, while the infinite loop of the impulse function is displaced unchanged along the time axis to $t = g$. For positive values of the parameter f, the cisoidal oscillation is a right-handed helix with pitch equal to f^{-1}, and thus decreasing as f increases. For negative values of f, the pitch is the same but the helix is left-handed.

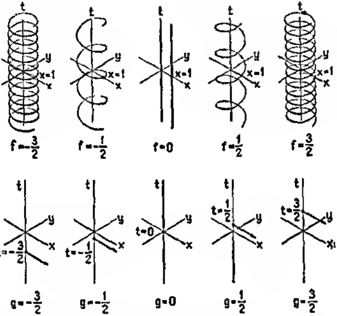

Fig 1—Wire models of cisoidal oscillations cis $(2\pi ft)$ (above) and of unit impulses $\mathfrak{S}_1(t-g)$ (below) for the particular values $0 \pm 1/2 \pm 3/2$ of the parameters f and g. Cf pairs (601) (982)

Both functions have essential singularities, which are quite different both in character and in location. For the cisoidal oscillation the singularity is always located at $t = \pm \infty$, for the impulse the singularity is at $t = g$.

The fundamental differences between the two elementary time functions adapt them for different uses. It is desirable to be in a position to employ first one and then the other, shifting from one to the other without any trouble or delay, so that at each step of a

problem the elementary function best suited for use may be employed. For this we require only an adequate table of pairs and a certain familiarity in the use of the pairs. It is desirable to acquire the habit of thinking of the coefficients of a pair as alternative representations of a curve.

The Use of Table I for Obtaining Fourier Integrals

The listed Fourier integrals are for the complex exponential $e^{i2\pi fg}$, but the integral sign and the exponential have been omitted to make the table more compact. Thus only two coefficients $F(f)$ and $G(g)$ appear in the table opposite a serial number for the pair. The integral sign and the exponential may be restored in connection with either coefficient, thus giving for each pair in Parts 1–9 the two Fourier integrals:

$$\int_{-\infty}^{\infty} F(f)e^{i2\pi fg}df = G(g),$$
$$F(f) = \int_{-\infty}^{\infty} G(g)e^{-i2\pi fg}dg.$$

From the table, the following sine and cosine Fourier integrals may also be written down at once:

$$\int_{-\infty}^{\infty} G(g)\cos(2\pi fg)dg = \int_{0}^{\infty} [G(g) + G(-g)]\cos(2\pi fg)dg$$
$$= \tfrac{1}{2}[F(f) + F(-f)],$$

$$\int_{-\infty}^{\infty} G(g)\sin(2\pi fg)dg = \int_{0}^{\infty} [G(g) - G(-g)]\sin(2\pi fg)dg$$
$$= i\tfrac{1}{2}[F(f) - F(-f)],$$

$$\int_{-\infty}^{\infty} F(f)\cos(2\pi fg)df = \int_{0}^{\infty} [F(f) + F(-f)]\cos(2\pi fg)df$$
$$= \tfrac{1}{2}[G(g) + G(-g)],$$

$$\int_{-\infty}^{\infty} F(f)\sin(2\pi fg)df = \int_{0}^{\infty} [F(f) - F(-f)]\sin(2\pi fg)df$$
$$= -i\tfrac{1}{2}[G(g) - G(-g)].$$

As explained above, it is most convenient to regard the table of Fourier integrals as a table of paired coefficients, and in the subsequent detailed description of the table this point of view will be maintained.

Summary of Table I

The table is divided into thirteen parts. The discussion that follows refers primarily to the first nine parts of the table. Parts 10–13 are

discussed later in separate sections In Part 1 are given the general processes for deriving any coefficient mate but such processes are to be employed only when it is necessary to start from first principles All mates which have once been determined may be taken from the latter sections of the table with a great saving of time and energy Part 2 of the table shows the elementary transformations and com binations of pairs these theorems may be employed either to extend a given table of coefficient pairs or to cover a given group of coeffi c nts with a shorter table of specific pairs It is assumed that anyone desiring to make serious use of the table will first become familiar with these elementary combinations and transformations even the simple addition factor and transposition theorems (201) (205) (217) are most useful

Part 3 of the table contains fif een pairs which are called key pairs because all specific pairs listed in Parts 4–9 of the table may be derived from them by specialization or by passing to a lim t after any necessary use has been made of only such elementary combinations and transformations of Part 2 as do not call for the evaluat on of an integration If all the pairs of Part 2 were employed the key I s. could be reduced to a single pair since any coefficient can in general be expressed as an integral involving any other specified coeffic ent The key pairs are listed in Part 3 with indications of the numbers assigned to them in the subsequent parts of the table but without repetition of the complete necessary and sufficient restrictions on the pa ameters involved For these restrictions reference must be made to the complete listings of the key pairs in their proper numerical order in Parts 4–9 of the table.

For applicational purposes it is most desirable to have a table which lists the precise pair required many special cases which have been used in practical applications may be found in Parts 4–9 of Table I which constitute a short classified list of particular cases It is important to remember that a given coefficient should be looked for on the other side of the table if it is not found on its own side since all pairs are transposable by (217) or (218) In the tables as they stand some pairs have been transposed but this is not true in the majority of cases

Whenever an infinite process is to be employed such as infinite series integration or differentiation the permissibility of the process is a question which must be answered for the particular case in hand, the formal result given in Table I may break down for example if either the original or the transformed pair is a s ngular pair This general warning necessarily applies to every part of the subject of coefficient pairs because it is a part of the general subject of mathematical analysis

It is intended that the statement of each pair in the table shall include the necessary and sufficient restrictions on all of its parameters to make the pair valid. First each letter is restricted to a definite domain, as listed in the table of notation; it is understood that this domain is applicable unless specifically modified. The symbol $\boxed{C}$ marks cases where this domain may be extended beyond that defined by the standardized notation. The symbol $\boxed{D}$ marks cases where this domain must be further restricted. Among the special cases listed under a pair, some are valid for the same domains of all the parameters as those of the pair; these cases are designated by $\boxed{S}$. Some admit extensions of the domains of some of the parameters; these cases are marked with $\boxed{B}$. Some require further restrictions on the domains of some of the parameters; these are marked with $\boxed{R}$. The necessary and sufficient restrictions on the parameters to give a valid pair in these $\boxed{B}$ and $\boxed{R}$ cases are found in the table under these particular pairs, and these restrictions are not included in the $\boxed{C}$ and $\boxed{D}$ conditions listed under the original pair. Singular pairs obtained as limiting cases of regular pairs are designated by $\boxed{L}$. Such limit pairs are listed in the table, but it is to be understood that these pairs are not obtained directly by integration, but that they are the limit approached by regular pairs. The method of approach is shown in every case for the $F(f)$ function.

In the actual work of deriving the special case where a parameter λ assumes the value λ_0 both coefficients have been multiplied by such a function of λ, and the approach of λ to λ_0 has been restricted to such a path, as will make both coefficients approach definite limits, neither zero nor infinite. Substitutions in the remaining parameters have sometimes been necessary to secure a valid pair upon taking the limit, and in particular to secure the individual pair referred to by number.

Having now explained, in a general way, the use of Table I, it will be useful to consider in detail a limited number of the pairs which are of special practical interest.

General Processes for Deriving the Mate

The table is naturally headed by the fundamental Fourier integral (101) because of its intrinsic importance as the explicit definition of coefficient mates. Pair (102) is then the expression of the Fourier integral theorem; it is thus of importance as the implicit definition of coefficient mates. The chief purpose of the table, however, is to make it possible for the technical man to make the fullest use of coefficient pairs without concerning himself at all as to the analytical work of evaluating either of these Fourier integrals. Pairs (101) and

(102) are thus intended to serve mainly as definitions for the pairs which follow in Parts 1-9

The statement has been made that essentially only one Fourier integral has been evaluated by determining the indefinite integral and substituting the integration limits Whether or not this is precisely true the statement does illustrate the fact that the formulation of the Fourier integral does not in itself suggest a practical finite analytical process for the actual evaluation of the definite integral No such system of evaluating definite integrals is known Writing down the Fourier integral amounts to little more than definitely formulating a question

If the coefficient $F(f)$ is expanded as a finite or infinite series in powers of f (or p) the mate is given by pair (106) and this involves a finite or infinite series of essentially singular functions which are further considered below in connection with Fig 3 If a series expansion of $F(f)$ is made in terms of any functions of f for which the mates are known there is a corresponding series for the mate. Some of these pairs are shown as (104) (112) The possibility of the formal infinite expansion does not necessarily imply the convergence of the series in the case of coefficient pairs any more than in other general developments

The technical man is not ordinarily a master of infinite series, definite integrals or other infinite processes It is therefore highly desirable to give him coefficient pairs which are in closed form that is involve only a finite number of operations with known functions. Accordingly the portion of the table expressible in closed form has seemed to be the part which should be developed first Specific pairs requiring infinite series for the expression of their coefficients have not been included in this revision of Table I A few Fourier series, however are listed in Part 10

The Elementary Transformations of Coefficient Pairs

The simple addition theorem (201) is of the greatest practical importance The summation may include any number of pairs they may be quite unrelated or they may be the successive terms of power expansions as shown in (106) (111) Next to the addition theorem we may place the multiplication theorem (202) or (203) special cases of which are of great practical importance Among these special cases are (206) (211) where any coefficient is multiplied or divided by its parameter or by a cisoidal oscillation of its parameter

Any real linear substitution for the frequency and epoch parameters is made possible by the simple transformations (205)-(207) (214)

The differentiation and integration of coefficients with respect to the frequency, epoch or other parameter give the important transformations (208)–(213).

Some of the simple transformations continue to yield new results when they are repeated any number of times or when several transformations are combined in sequence. Pairs (216), (218)–(222) are examples of such combinations.

The resolution of pairs into the four types of i^n-multiple pairs, as shown by pairs (223)–(225), throws considerable light on the nature of coefficient pairs.

Some of the elementary properties of pairs are expressed *in words* as follows:

ELEMENTARY PROPERTIES OF PAIRS

(1) The sum or difference of pairs is a pair. Cf. pair (201).

(2) Any constant multiple of a pair is also a pair. Cf. pair (204).

(3) Any linear combination of pairs is also a pair. Cf. pairs (201), (204).

(4) The odd and even parts of every pair are also pairs.[4a]

(5) If both coefficients of a pair are real, both are even.

(6) If a pair has one real and one pure imaginary coefficient, both are odd.

(7) If a coefficient is even and real, its mate is also even and real.

(8) If a coefficient is odd and real, its mate is odd and pure imaginary, and vice versa.

(9) If a coefficient is real, its mate has conjugate values for opposite values of its parameter and conversely. Cf. pair (216).

(10) The conjugates of the coefficients of a pair are also a pair provided the sign of either frequency f or epoch g is reversed. Cf. pair (215).

(11) A pair with the signs of both frequency f and epoch g reversed is also a pair. Cf. pair (214).

(12) The parameter of either coefficient may be multiplied by a positive real constant provided the other parameter and coefficient are each divided by the same constant. Cf. pair (205).

(13) Coefficients of a pair may be interchanged if, when interchanging the parameters, the sign of one parameter, either f or g, is reversed. Cf. pair (217).

(14) Any pair may be resolved uniquely into the sum of four pairs by pairing together: the even, real parts; the even, imaginary parts; the odd, real part of each coefficient with the odd, imaginary part of the other coefficient.

[4a] Some pairs may be partitioned into any number of pairs by Simpson's method (Chrystal, Algebra, II, p. 416) as is illustrated by the pairs of Part 10.

(15) A pair may have the form $(F(f), \lambda F(g))$, where the multiplier λ is constant if and only if λ has one of the four unit values $(1, i, -1, -i)$ Such a pair is called an i^n multiple pair Cf pair (223)

(16) Any i^n multiple pair has both coefficients odd or even according as n is odd or even

(17) Any i^n multiple pair with complex coefficients may be resolved into two i^n multiple pairs with coefficients which are real or pure imaginary

(18) The coefficients of any two i^n multiple pairs are orthogonal if the i^n multipliers are different.

(19) The coefficients of any four i^n multiple pairs with different i^n multipliers are linearly independent

(20) Any pair may be resolved uniquely into the sum of four i^n-multiple pairs i e pairs of the form $F_n(f)$ $i^n F_n(g)$ Cf pair (224)

(21) Any pair may be resolved uniquely into the sum of eight i^n-multiple pairs where $F_n(f)$ is real or pure imaginary Cf pair (225).

Pairs Based on the Normal Error Law

The identical pair (705 1), $\exp(-\pi f^2)$, $\exp(-\pi g^2)$, is one of the simplest pairs and may well serve as the starting point in the consideration of specific coefficient pairs Each coefficient is the impulse of the normal error law It is remarkable that identical coefficients of this simple form should produce the same identical function when associated with either the cisoidal oscillation or the very different unit impulse

If the differential transformation (222), taking the upper signs, is applied to the normal error law pair (705 1) expressed in the alternative form (704 0), the infinite series of ϕ_n pairs (702) is obtained Of these derived pairs the first nine are written out as pairs (704 1)–(704 9) The cisoidal coefficients are alternately even and odd functions which oscillate in the neighborhood of the origin, each successive coefficient having an added half oscillation The ϕ_n pair has $(n+1)$ half oscillations Beyond these oscillations every coefficient in the infinite sequence decreases rapidly and asymptotically to zero in both directions The mates of these cisoidal coefficients are identically the same except for a constant coefficient which is i^n and thus goes cyclically through the four values, $1, i, -1, -i$

The $\phi_n(x)$ functions are shown by Fig 2 They are essentially the parabolic cylinder functions of order n These coefficients may be used for the expansion of every function which, with its first two derivatives, is continuous for all positive and negative values of the variable and for which a certain integral exists This expansion known as the Gram Charlier series, appears in pair (112)

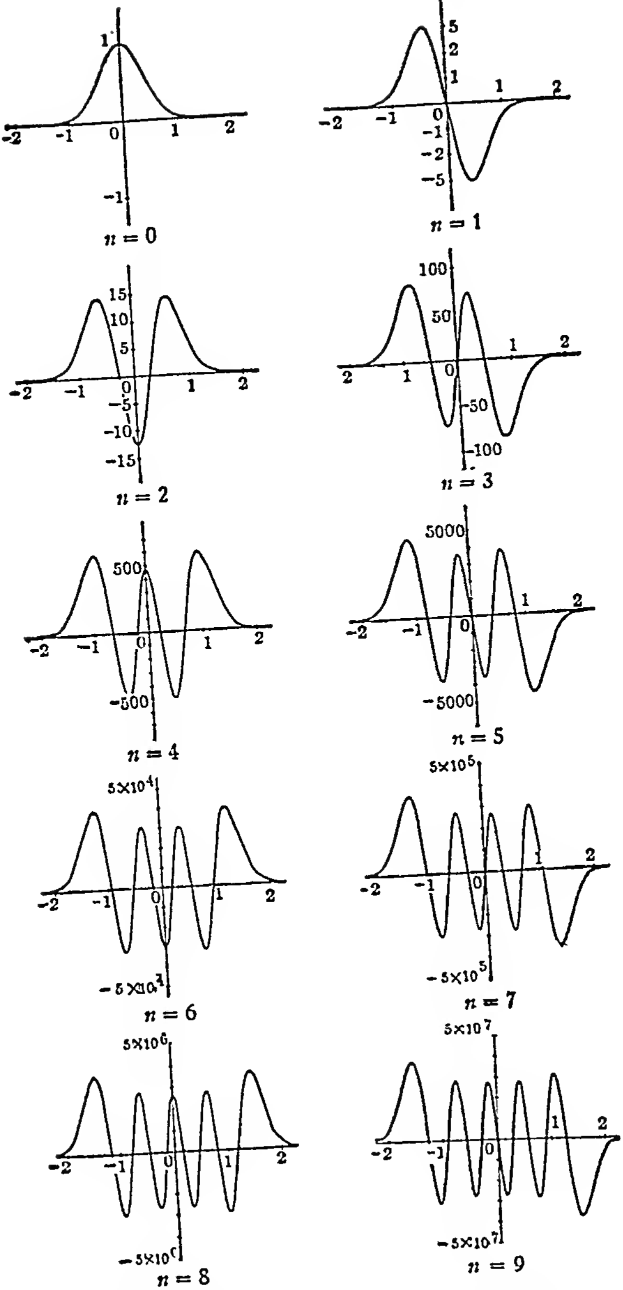

Fig. 2—Curves showing the $\phi_n(x)$ functions for $n = 0, 1, 2, \cdots, 9$.
Cf. pairs (704.0)–(704.9).
$\phi_n(x) = \exp(\pi x^2) D_x{}^n \exp(-2\pi x^2)$.

Starting again with the normal law of error pair (705 1) in the form (710 0) and setting $\rho = \frac{1}{2}\beta/\pi$ and applying the differential transformation (208) repeatedly, we obtain the infinite sequence of pairs (707 0) of which the first five are listed as pairs (708 0) (708 4) The cisoidal coefficients are the successive integral powers of p multiplied by the normal error exponential The impulse coefficients are essentially the ϕ_n functions multiplied by the normal error exponentials These pairs are plotted in Fig 3 for the special case $\beta = a^2 = 1$

Both of the infinite series of pairs derived from the error function and shown in Figs 2 and 3 are regular throughout are nowhere infinite and vanish at infinity Many other series of pairs might be built up from the error function pair In the table the identical pairs (706 1)-(706 6) have been included these are of interest in that the polynomial multipliers are all perfect squares These coefficients may be expressed as linear combinations of the functions ϕ_{1n} ϕ_{4n} ϕ_4 ϕ_0 The χ_n functions of pairs (706 7)-(706 74) constitute another set of functions which are linear combinations of the even functions ϕ_{2n}, ϕ_{2n-4}

Essentially Singular Pairs for Integral Powers of the Parameter

If in Fig 3 with the value of n held fixed, we allow a to approach the limit 0 the cisoidal coefficient becomes p^n and the impulse coefficient which is compressed horizontally towards the origin and expanded vertically with corresponding areas increasing as a^{-n}, ultimately vanishes everywhere except at the origin where it acquires an essential oscillating singular point At the limit then a singular pair is obtained it will be designated as p^n, $\mathfrak{S}_n(g)$ $\mathfrak{S}_n(g)$ is characterized by having all of its moments about the origin vanish except the nth moment, which is equal to $(-1)^n n!$ The dotted graphs on the left of Fig 3 show p^n to the scales indicated The curves on the right show $\mathfrak{S}_n(g)$ provided we assume that the horizontal scale is increased with a and the vertical scale increased inversely with a^{n+1} as a approaches the limit 0 Fig 3 thus serves to picture the essentially singular function $\mathfrak{S}_n(g)$ That is it is sufficient if the coefficient maintains this form while proceeding to the limit This form is however, not essential[5] It is apparently necessary only that the method of approach to the limit give the same set of moments

An alternative way of deriving the mate for the positive integral powers p^n is by means of a linear combination of $(n + 1)$ pairs of the

[5] A quite different treatment of singular functions has recently been given by W E. Sumpner Impulse functions *Phil Mag.*, (7) 11, 345 368 (Feb 1931)

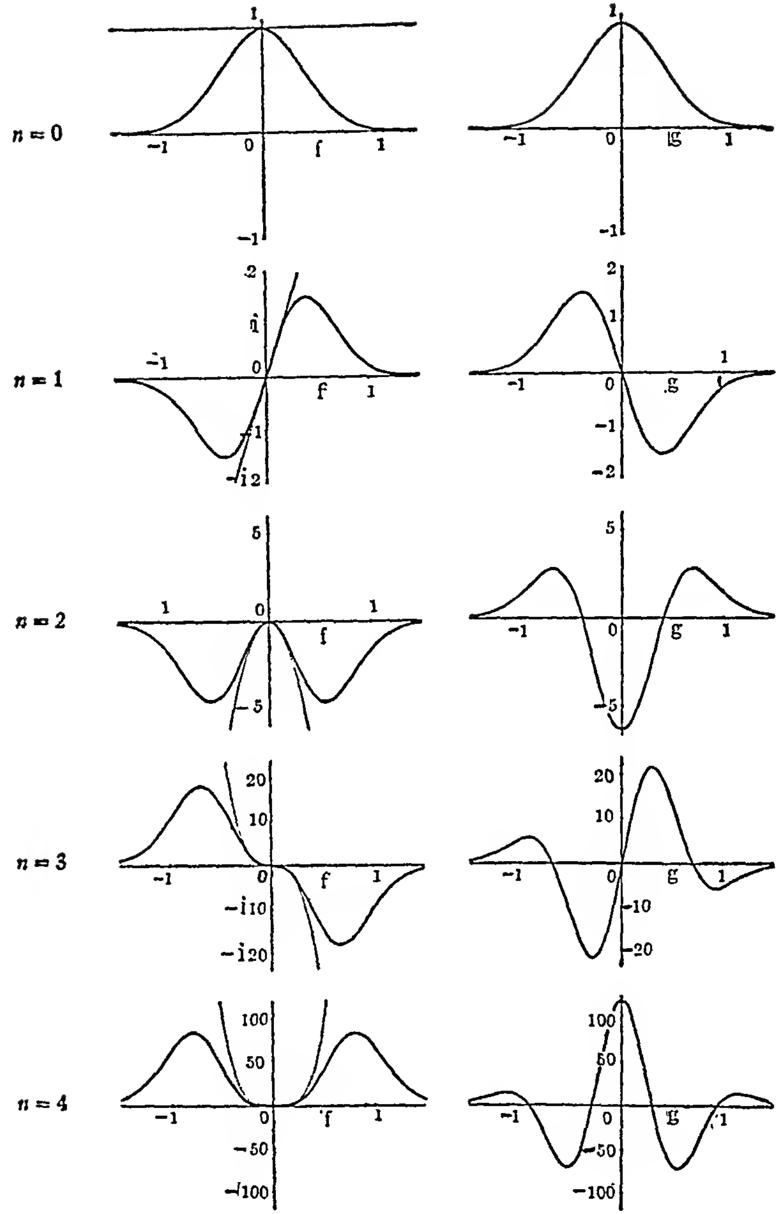

Fig. 3—Graphs for the family of pairs $p^n \exp(-\pi a^2 f^2)$, $a^{-1}D_0^n \exp(-\pi g^2/a^2)$. The heavy curves show the cases $a = 1$, $n = 0, 1, 2, 3, 4$; the dotted curves on the left are for the same values of n but for the limit $a \to 0$. On the right the curves apply for any value of a if the horizontal and vertical scales are multiplied by a and a^{-n-1} respectively. Cf. pairs (401.1)–(404.1), (708.0)–(708.4).

form of (603) with parameters equal to a, $2a$ $3a$ $\cdot$, $(n+1)a$, respectively, so that the first term in the power series expansion of the cisoidal coefficient is p^n The corresponding impulse coefficient is a succession of $(n+1)$ bands each of width a, the first band beginning at epoch zero the heights of the successive bands being equal to the binomial coefficients for power n divided by a^{n+1} but alternately positive and negative The mth moment of this impulse coefficient is 0 for $m < n$ equal to $(-1)^n n!$ for $m = n$, and proportional to a^{m-n} for $m > n$ Upon allowing a to approach zero the cisoidal coefficient approaches p^n, and the impulse coefficient approaches $\mathfrak{S}_n(g)$ since in the limit the same set of moments is obtained as was found above to characterize the nth singularity function This is pair (402)

The special cases for $n = 0$ 1 are of most frequent occurrence They are pairs (403 1) (404 1) $\mathfrak{S}_0$ is the unit impulse since its 0th moment equals unity, $\mathfrak{S}_1$ is the doublet with the moment -1 since its first moment is -1 $\mathfrak{S}_1$ and all higher order singular functions are included in the series coefficients of (104) (106)

Fig 3 may be extended upward step by step from the normal error law pair by dividing by p on the left and integrating with respect to g on the right At each step a constant of integration is introduced The first two pairs thus obtained are pairs (725 1) and (726 1) Choosing the integration constants so as to make the impulse coefficients alternately odd and even these two pairs are as shown in Fig 4 If we now allow a to approach the limit zero a new series of pairs is obtained of which the first two pairs are shown dotted in Fig 4 for the particular choice of integration constants there made The general limiting pair is designated as p^{-h} $\mathfrak{S}_{-h}(g)$ and it is shown as pair (410) In some ways it is simpler to derive the limiting pair for negative integral powers of p from rational functions of p which may be accomplished as shown by pair (411 1) Special cases are shown by pairs (408 1), (415) (416)

The first of the series $\mathfrak{S}_{-1}(g)$ is a unit step at epoch 0 from a constant value $\lambda - \frac{1}{2}$ for all negative epochs to the constant value $\lambda + \frac{1}{2}$ for all positive epochs The constant λ may have any value, this is a singular case marked by the failure of the general rule that the choice of the cisoidal coefficient uniquely determines the impulse coefficient This means that in any well set problem some other condition determines the value of the constant λ In some problems for example it is necessary that the epoch coefficient be an odd function and then λ vanishes In other problems where either the epoch function must be zero for all negative epochs or on the other hand the p occurring in the cisoidal coefficient is actually the limit of $p + a$ as a approaches zero through

positive values, the constant λ equals $\frac{1}{2}$. This limiting condition may arise if we assume that resistance may be ignored, as a first approximation, in studying actual systems which necessarily involve at least a small amount of dissipation.

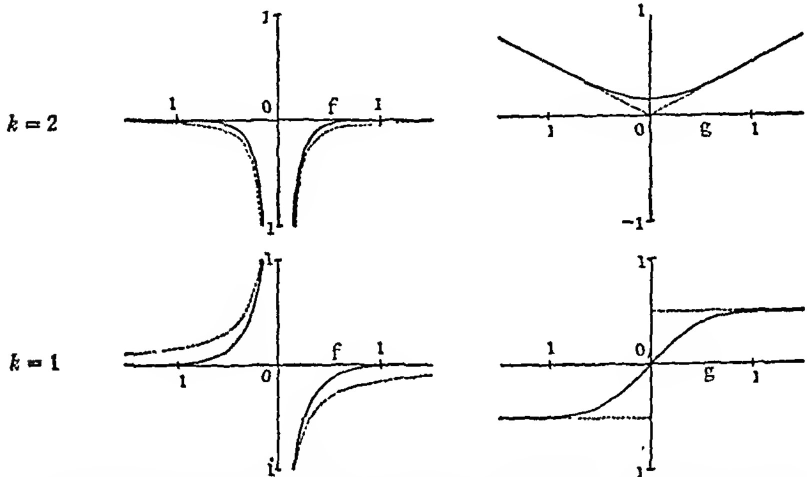

Fig. 4—Graphs for the family of pairs $p^{-k}\exp(-\pi a^2/f^2)$, $a^{-1}D_g^{-k}\exp(-\pi g^2/a^2)$, with the integration constants chosen so as to make the impulse coefficients alternately odd and even. The heavy curves show the cases $a = 1, k = 1, 2$; the dotted curves show the limit $a \to 0, k = 1, 2$. Cf. pairs (410), (415), (416), (725.1), 726.1).

The mates of positive and negative integral powers of p, including the zero power, cannot be derived directly and definitely from the Fourier integral (101) without the specification of an additional passage to a limit. Such pairs therefore differ essentially from the great body of regular pairs where the choice of one coefficient completely determines the mate. These pairs may be thought of as lying on the periphery of the great domain which includes the totality of regular pairs.

Identical Mates and Other Simply Related Mates

Since one of the coefficients of a pair may be assigned quite arbitrarily, this choice allows us, if we so elect, to specify some relation between the two coefficients of a pair. We might specify that a linear combination $\lambda F_j(x) + \mu G_j(x)$ of the two coefficients of a pair both taken with the parameter x is to equal an arbitrary function $F(x)$. The pair (F_j, G_j) is then uniquely determined, unless $\lambda + i^n\mu = 0$, being equal to pair (224) after each F_n has been divided by $\lambda + i^n\mu$. Again if it is specified that one coefficient is to be the reciprocal of the other, a possible solution is pair (760).

The condition that the mates shall be identically the same function of the r parametric variables f and g is of special interest.[4] In addition to the identical pairs shown on Fig. 2, $n = 0.4.8$, the table contains a number of identical pairs including (523), (625), (706.1), (706.6), (761), (916), (916.3), (923.85), (966.25).

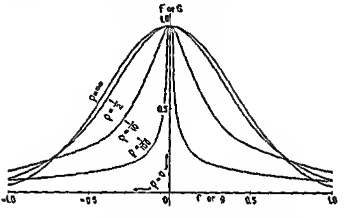

Fig. 5—Identical coefficient pairs of the form
$(1 + x^2/\rho^2)^{-\frac{1}{2}}K_1(2\pi\rho^2\sqrt{1 + x^2/\rho^2})/K_1(2\pi\rho^2)$ $\quad x = f$ or g
Cf. pairs (916), (705.1), (523)

The identical pair (916) divided by its value at the origin is shown in Fig. 5 for different real values of its parameter ρ. For $\rho = +\infty$, the curve is of the $\exp(-\pi x^2)$ or normal law of error form and is identical pair (705.1). For $\rho = \frac{1}{2}$ the reciprocal hyperbolic cosine identical pair (625) is shown correctly within the width of the line, this being apparently a mere coincidence since pair (916) does not include it as a special case. Finally for $\rho = 0$ the limiting curve coincides with the horizontal axis taken together with unit length of the positive vertical axis. This represents pair (523) divided by its value at the origin, which is infinite. The point to be especially noted is that the area under every curve of the family illustrated by Fig. 5 is the same and equal to unity. This must hold for the limit $\rho = 0$ when the curve encloses no area within a finite distance of the origin.

[4] Identical pairs are closely related to functions which are self reciprocal in the Fourier cosine transform. Such functions have been treated by W. N. Bailey, Some classes of functions which are their own reciprocals in the Fourier Bessel integral transform, *Jour. London Math. Soc.* 5, 258-265 (Oct. 1930); also G. H. Hardy and E. C. Titchmarsh, Self reciprocal functions, *Quart. Jour. Math.* (Oxford) 1, 196-231 (Oct. 1930).

The identical pair $|f|^{-\frac{1}{2}}$, $|g|^{-\frac{1}{2}}$ is of great simplicity and it occupies a central position among algebraic pairs. Starting with the minus one-half power of the parameters in both coefficients, any increase in the power of one parameter requires an equal decrease in the power of the other parameter as is illustrated, for example, by pairs (502), (516), (524). The identical pair (916.3) is of considerable interest in that both coefficients are identically zero for all values of f or g less in absolute value than the arbitrary parameter r. The identical pair (761) is shown in Fig. 6, and the reversed pair (762) in Fig. 7.

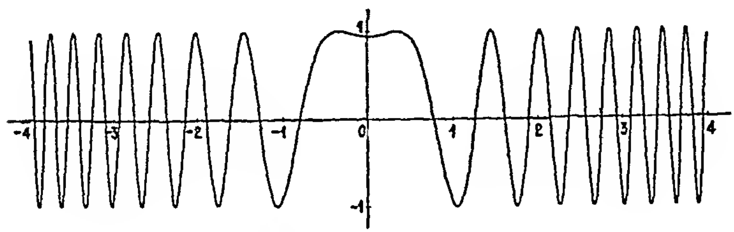

Fig. 6—Coefficient cos $[\pi(x^2 - \frac{1}{8})]$ of identical pair (761), $x = f$ or g.

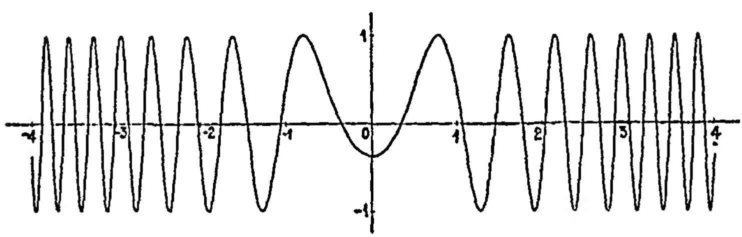

Fig. 7—Coefficient sin $[\pi(x^2 - \frac{1}{8})]$ of reversed pair (762), $x = f$ or g.

It is not permissible to specify any relation whatsoever between the two coefficients of a pair; for example, no pair exists for which one coefficient is twice the other. As stated above, the only multiples permissible are the four units $1, i, -1, -i$. For each of these four cases there are an infinite number of solutions. These solutions satisfy the integral equations given in the foot-note to pair (223).

Fourier Series

Fourier series appear in Part 10 of Table I as limits of Fourier integrals. The $G(g)$ coefficient is taken periodic with the period 2π. The $F(f)$ coefficient limit is then zero everywhere except at the equispaced points corresponding to $2\pi f = p/i = v = 0, \pm 1, \pm 2, \cdots$ where it may have so approached infinite limits as to cover finite areas. These areas are given in the $F(f)$ column. The general expression

for the areas is usually preceded by specific values for $v = 0, 1, 2, \cdots$ which illustrate the general expression, in some cases the first few specific values do not agree with the general expression and replace it When non vanishing finite areas do not occur for negative values of v, the fact is emphasized by replacing the parameter v by n Multiplying the area at the point corresponding to v by $e^{i\nu\tau}$ and adding all terms gives the Fourier series so that the areas serve as an abbreviated expression for the series

Cosine and sine series are obtained by equating the real and imaginary parts of the series and of $G(g)$ This partition is not carried out on the $F(f)$ side in the table, but only on the $G(g)$ side Another simple partition of the series is into its odd and even terms, the partition of $G(g)$ into its odd and even parts is shown, also the partition of these odd and even parts into their real and imaginary parts To visualize the general case of this type of partitioning, we may think of the series as arranged in a rectangular array of k rows with the first k terms of the series ($v = 0$ 1, 2, , $k - 1$) in the first column of the array, beginning at the top, the next k terms in the second column, and so on indefinitely Opposite the k rows are placed the corresponding partitionings of $G(g)$ as given by Simpson s method Then the series in any row taken alone is equal to the part of the $G(g)$ function in the same row Each Fourier series thus obtained may be broken up into cosine and sine series by equating real and imaginary parts In the table, the areas occurring in the $F(f)$ column have not been placed in arrays with two, three, four, etc rows because such arrays are easily visualized, but the corresponding partitioning of $G(g)$ has been carried out, successive terms being given in sequence although not always on separate lines In some cases each term is divided into its real and imaginary parts When this has been done, the imaginary part follows the real part and it begins with the imaginary symbol i

Only a few Fourier series have been listed, merely enough to make a trial of this method of giving important partitionings of $G(g)$ in place of listing these related series separately

Contour Integrals

The first nine parts of Table I are concerned exclusively with integration along the entire real axis of f, and these integrals occupy a preferred position because they may be transposed by the simple Fourier transposition of pair (217) In the synthesis of complex exponentials, there is, however, no necessity for restriction to the real axis and for certain practical applications, contour integrations in the

complex f plane may present distinct advantages. For this reason, Table I has been extended tentatively to include a few contour integrals. The contour integrals which have been assembled for this purpose are divided into three groups, according as the path of integration in the f plane is a parallel to the real axis, a closed contour, or a path with arbitrary end points.

Part 11 may be regarded as a tabulation of Bromwich's generalized Fourier integrals, where the path of integration in the f plane is a straight line parallel to and at the distance $x/(2\pi)$ below the real axis, with all singularities of the integrand, regarded as the function of f, lying above this path. In every case, however, such an integral can be reduced to the Fourier integral of the earlier parts of Table I by the relation:

$$G_1(g) = \int_{-\infty - ix/(2\pi)}^{+\infty - ix/(2\pi)} F_1(f) e^{i2\pi fg} df = e^{xg} \int_{-\infty}^{\infty} F_1\left(f - \frac{ix}{2\pi}\right) e^{i2\pi fg} df.$$

There is complete equivalence, therefore, between a pair $F_1(f)$, $G_1(g)$ in Part 11 and a pair $F_1\left(f - \dfrac{ix}{2\pi}\right)$, $e^{-xg}G_1(g)$ in Parts 1–9. If it is permissible to take $x = 0$, then for this special case the two expressions for the pair become identical. Under each pair in Part 11, the corresponding pair of an earlier Part is listed opposite the $\boxed{A}$ symbol.

Part 12 contains contour integrals along a closed path, finite or infinite. The parameters are unrestricted, although when the contour extends to infinity, it may be necessary to consider more than one asymptotic direction in order to obtain all sectors in the G plane. By elementary transformations and combinations (not involving any integrations) all pairs of Parts 11 and 12 can be obtained from the four pairs (1264)–(1267). The Fourier integral transformation theorem, pair (217), does not apply to these contour pairs generally and the special information required for transposition is not listed in the table. Since the parameters are unrestricted, closed contour integrals present special advantages and a more complete tabulation is desirable.

Part 13 gives a few indefinite integrals which may be used with any arbitrarily assigned upper and lower limit points for the path of integration—provided due regard is paid to the singular points. In this part of the table, f and g are unrestricted complex variables. The functional relations (1301)–(1307) may be used to build up other indefinite integrals from those tabulated, and the following pairs of Part 2 are also applicable: (201), (204), (208), (210), (212) and (213). The transposition theorem, pair (217), does not apply directly. Using only the elementary transformations and combinations which do not require integration, the specific integrals of this part can all be obtained from the three pairs (1312), (1317) and (1320).

Practical Applications of Coefficient Pairs

Fourier gave the first comprehensive method of finding the solution for transients His method involves three steps viz,

I Spectrum analysis of the cause among all frequencies
II Solution for all frequencies
III Spectrum synthesis of the effects for all frequencies

Fourier thus substituted three problems for one With a table of Fourier coefficient pairs these three steps may be made as follows

I Find the mate of the cause considered as an impulse coefficient
II Multiply this mate by the admittance for the system
III Find the mate of this product considered as a cisoidal coefficient

These three steps define a perfectly definite result since every arbitrarily chosen coefficient has a mate which is unique and determinate, or may be made so by the specification of some suitable passage to a limit

The use of a table of pairs may also be stated in another and somewhat more general way as follows

For any system where the principle of superposition holds any cause $C(g)$ its effect $E(g)$ and the corresponding admittance $Y(f)$ are connected by a relation which may be written in any one of three ways which explicitly express each of the three quantities in terms of the remaining two as follows

$$E(g) = \mathfrak{M}[Y(f)\mathfrak{M}C(g)]$$

$$C(g) = \mathfrak{M}\left[\frac{\mathfrak{M}E(g)}{Y(f)}\right]$$

$$Y(f) = \frac{\mathfrak{M}E(g)}{\mathfrak{M}C(g)},$$

where $\mathfrak{M}$ is read 'mate of'

This general relation between transients and the Fourier coefficient pairs is repeated in the following tabulation together with the three special cases which are discussed later in connection with Table II

Admittance $Y(f)$ or Operation $\phi(p)$	*Impressed Driving Cause $C(g)$*	*Transient $E(g)$*	*Based on Pair No*
$F_2(f)/F_1(f)$	$G_1(g)$	$G_2(g)$	
$F(f)$	$\mathfrak{S}_0(g)$ = unit impulse	$G(g)$	403 1
$pF(f)$	$\mathfrak{S}_{-1}(g)$ = unit step	$G(g)$	415
$(p - p_0)F(f)$	$e^{i2\pi f_0 g}$ beginning at $g = 0$	$G(g)$	440

Thus Table I just as it stands, with t substituted for g, presents for an assumed admittance $F(f)$ the transient $G(t)$ which would be produced by a unit impulse acting at time $t = 0$. Again, after multiplying each coefficient in the F column by p, the admittance and transients are for the unit step acting at time $t = 0$. And, generally, since the F and G coefficients may be transposed by pair 217 any coefficient in Table I may be used as the transient effect $E(t)$ due to this or any other coefficient acting as the driving cause $C(t)$. This merely calls for an admittance equal to the ratio of the coefficients paired with $E(t)$ and $C(t)$. Whether there is any known physical phenomenon in a physical system corresponding to this mathematical transient is an open question. It is answered positively in a number of cases by the entries in Table II.

In the operational method of discussing transients, the admittance $Y(f)$ is referred to as the operator $\phi(p)$. By giving the first column above both headings, the tabulation applies equally well to both the Fourier integral method and the operational method. It is assumed, of course, that in both methods, all limits are approached in manner and sequence to correspond with the physical phenomenon. For example, in certain series expansions, it may not be permissible to treat as real both the variable p in $\phi(p)$ and f in $Y(f)$.

The use of coefficient pairs may be most simply illustrated by reference to Figs. 3 and 4, in connection with the problem of finding transient currents through a perfect condenser of unit capacity, due to impressed electromotive forces shown by each of the seven curves on the right, considered as functions of the time. Any curve on the right being the cause, the next curve below it is the effect, considering Fig. 4 to be placed above Fig. 3. In the solution the first step is to find the mate of the curve on the right. This is the curve on the left. This mate is then to be multiplied by the admittance of the system, which is p for a unit condenser. Reference to the titles of the figures shows that this product is given by the next lower curve on the left. To find the mate of this last curve is the third step in the solution and for this it is merely necessary to go to the curve on the right. The three steps then take us from any curve on the right to the next curve below it. Figs. 3 and 4, taken together, are a section of an infinite sequence of pairs which illustrate an infinite number of possible transients in a perfect condenser of unit capacity.

If, on the other hand, the system consisted of a perfect reactance coil of unit inductance and the impressed cause was again shown by any curve on the right, the effect would be shown by the next higher curve, assuming that the initial current at the beginning of time was

that shown by the extreme left of the upper curve Thus, when the cause is oscillating there is one less half oscillation in the effect than in the cause This is for an inductance For a condenser conditions are reversed the effect has one more half oscillation than the cause

The scales of Figs 3 and 4 may be changed to correspond to any value of a the parameter which appears in the coefficients of the pairs At the limit $a = 0$ the cause and effect would be the singular $\mathfrak{S}_0$ or $\mathfrak{S}_{-1}$ functions

The curves on the right for $n = 0$ of Fig 3 and $k = 1$ of Fig 4 show that at the limit $a = 0$ a unit step in the voltage produces a unit impulse in the current through a unit condenser on the other hand a unit impulse applied to a unit inductance gives a current which is a unit step

The curves of Fig 2 may be used to furnish another illustration of the use of coefficient pairs in connection with the problem of finding networks in which assigned transient currents will be produced by assigned impressed electromotive forces Any curve n being the assumed cause and the next curve $(n + 1)$ the assumed effect the required admittance is $\phi_{n+1}(f)/[i\phi_n(f)]$ This admittance is presented by a ladder network of $(n + 1)$ elements perfect inductance coils in the series arms perfect condensers in the shunt arms the ladder starting with a shunt condenser the values of the shunt capacities being equal to 2 $2n(n-1)^{-1}$ $2n(n-1)^{-1}(n-2)(n-3)^{-1}$ etc and the values of the series inductances being equal to $(2\pi n)^{-1}$, $(2\pi n)^{-1}(n-1)(n-2)^{-1}$ etc In verifying the solution of this problem it is to be noticed that the mates of the curves n and $(n + 1)$ regarded as impulse coefficients are the same curves multiplied by s^{-n} and $s^{-(n+1)}$ the quotient of the latter mate divided by the former mate is the admittance of the network as given above

On the other hand any curve $(n + 1)$ being the cause the curve n is the effect in the reciprocally related ladder network of $(n + 1)$ elements starting with a series reactance coil the values of the series inductances being equal to 2 $2n(n-1)^{-1}$ $2n(n-1)^{-1}(n-2)(n-3)^{-1}$ etc and the values of the shunt capacities being equal to $(2\pi n)^{-1}$, $(2\pi n)^{-1}(n-1)(n-2)^{-1}$ etc

Applications of Coefficient Pairs in Table II

In general each of the three subsidiary problems employed by Fourier is unsolvable in closed form. In a strictly limited number of cases however all three problems have been solved and the final transient solution obtained These solutions should be cherished and collected for ready reference It is a needless waste of time to repeat the analytical work each time a solution is required Except for a

few special cases lying outside of the scope of the table, all practical applications of closed form coefficient pairs which were found in a preliminary search are included in the transient solutions of Table II. As it stands, the table is far from a complete list of closed form solutions, but it contains many important solutions and serves to illustrate the use of Table I. Table II contains 39 admittances, with references to 39 systems which serve to illustrate the occurrence of these admittances. In the third, fourth and fifth columns, 85 transient solutions are given of which 39 are for the unit impulse, 30 for the unit step, and 16 for the suddenly applied cisoid.

The causes producing the transients in Table II are but three in number: the unit impulse, the unit step, and the suddenly applied cisoid; and the mates for these causes are unity, p^{-1} and $(p - p_0)^{-1}$ as is shown by pairs (403.1), (415) and (440). Multiplying these three mates by the admittances and taking the mates of the products, we have the effects, as stated in the headings of the last three columns of the table.

To illustrate in detail the steps involved in finding a transient effect with the aid of Table I, consider system No. 14 of Table II with the cause equal to the unit step $\mathfrak{S}_{-1}(t)$, $\lambda = \frac{1}{2}$. The mate of the unit step is p^{-1} by pair (415). Multiplying this by $Y(f)$ as given in the second column of Table II, we have $up^{-1}(1 + \sqrt{p/\lambda})^{-1}$ for the cisoidal coefficient. By pair (551) the mate of this is $u\sqrt{\lambda}\exp(\lambda g)$ erfc $\sqrt{\lambda g}$, $0 < g$. Substituting for g the actual variable t, we have the transient solution as given in the fourth column and fourteenth row of Table II.

This simple example fully illustrates the three essential steps in finding any transient effect when the admittance and pairs are known. In this example the effect was considered to be the unknown. If either the cause or the admittance were the unknown, the same pairs would be involved but the two coefficients in a pair would be used in the reversed sequence in all but one instance.

There are still 32 squares of Table II left blank. It would be a simple matter to place series solutions or integral solutions in each of these squares. Thus if the impulse transient of column 3 is known, the other two transients are given at once in integral form by pairs (210) and (219); if the unit step transient of column 4 is known, the suddenly applied cisoidal transient is written immediately in integral form by the use of pair (220). The real problem is, however, either to find closed form solutions in terms of known functions or to show that this is impossible. When the failure of known functions has been established, we should next consider the choice of new functions so defined as to throw as much light as possible on the new solutions.

Table II may be regarded as another table of coefficient pairs. Column 2 contains cisoidal coefficients, column 3 the mates of these coefficients, column 4 the mates of these coefficients when multiplied by p^{-1} and column 5 the mates of these coefficients when multiplied by $(p - p_0)^{-1}$. The corresponding pair in Table I is referred to in the lower left hand corner of each square by its serial number. In a few cases, two or three pairs are referred to and there it is necessary to add the Table I pairs together or, in the case of systems 37–39, to apply the two pairs in sequence.[7] In Table II the customary physical notation is adhered to because it is often of long standing and this necessitates some change in notation when comparing pairs in the two tables.

Summary and Conclusions

A large number of the known closed form evaluations of Fourier integrals have been compiled and tabulated in conveniently compact form. Many practical applications of the Fourier integral have been simplified by the compilation of Tables I and II which give coefficient pairs, admittances and transient solutions.

Minor changes in nomenclature and point of view have been introduced, all with the idea of simplifying the practical application of the Fourier integral, in the following ways:

(1) Using the cisoidal oscillation and the unit impulse side by side as alternative elementary expansion functions.

(2) Focusing attention upon coefficient pairs for these two elementary functions, both coefficients of a pair representing the resolution of the same arbitrary function.

(3) Using the frequency and epoch as the parametric variables in place of the customary radian frequency and independent time variable.

(4) Employing as a coefficient any real or complex arbitrary function which may be practically useful by regarding it, where necessary, as a limit approached through coefficients which form regular pairs.

(5) Introducing the $\mathfrak{S}_n(g)$ functions having an essential oscillating singularity at the origin which mate with p^n, the positive integral powers of p.

(6) Using a notation which greatly reduces the number of occasions for employing the integral symbol in applications of the Fourier theorem.

[7] Some questions relating to operational methods involving two variables have recently been discussed by Balth. van der Pol and K. F. Niessen, "On simultaneous operational calculus," *Phil. Mag.* (7) 11, 368–376 (Feb. 1931). See also a paper by Marion C. Gray, "Note on some self reciprocal functions in the double Fourier transform," to be published in the *Journal of the London Mathematical Society*.

Having established the inclusiveness and practical utility of the proposed coefficient pair method of applying the Fourier integral, we are now planning to critically verify the tables and make them as complete as is feasible. It is proposed to include eventually such references to the literature [8] as may add to the interest of the tables. The contributions of integral equations and of the operational method to the present subject will also be incorporated in the tables. The preparation of similar tables for other elementary expansion functions, such as Bessel functions, is also a possibility. A comprehensive table might be made which would include in parallel columns the coefficient functions for a large number of elementary expansion functions, thus giving at once many alternative ways of representing particular time functions. This would make it possible to shift without trouble from any one expansion to any other expansion of the tabulation.

We are under great obligations to our colleagues for their contributions towards the preparation of this paper. While the primary task has been that of compiling the known compact evaluations of Fourier integrals and their systematic tabulation in a uniform notation, there has been, in addition, a great deal of checking and independent determination of parameter domains. Generalizations have also been made in certain integrals and in some of the tabulated applications.

We shall be grateful to any persons calling our attention to errors or omissions in the paper since we hope eventually to make the compilation of integrals and applications still more adequately meet the needs of those who make practical use of Fourier integrals.

[8] Most of the integrals listed in Table I may be derived from results presented in the following works: D. Bierens de Haan, "Nouvelles Tables d'Intégrales Définies," Leiden, 1867; T. J. I'a Bromwich, "An Introduction to the Theory of Infinite Series," second edition, London, 1926; G. N. Watson, "A Treatise on the Theory of Bessel Functions," Cambridge, 1922; E. T. Whittaker and G. N. Watson, "A Course of Modern Analysis," third edition, Cambridge, 1920. Operational equations have been presented by J. R. Carson, "Electric Circuit Theory and the Operational Calculus," New York, 1926 (revised German edition, "Elektrische Ausgleichsvorgänge und Operatorenrechnung," Berlin, 1929); by Harold Jeffreys, "Operational Methods in Mathematical Physics," Cambridge, 1927; and by Vannevar Bush, "Operational Circuit Analysis," New York, 1929.

The following notation is employed in Tables I and II except as specifically restricted

a, b, c	=	positive reals, infinity being excluded
br x	=	branch x of a multiple valued function. For algebraic functions branches are designated for any power of a variable in the manner listed under z^{ν}, br x below. For transcendental functions branches are determined by the corresponding branches of the arguments of the functions. When no branch designation is given branch zero is to be understood
$\operatorname{cis}(z)$	=	$\cos z + i \sin z = \exp(iz) = e^{iz} = i^{2z/\pi}$
	=	cisoidal oscillation if $z = 2\pi ft$
$C(z)$	=	$\int_0^z \cos(\tfrac{1}{2}\pi z^2)dz = -C(-z)$ $\quad C(\pm\infty) = \pm\tfrac{1}{2}$
$D_\nu(z)$	=	parabolic cylinder function of order ν
	=	$2^{\frac{1}{2}\nu+\frac{1}{4}} z^{-\frac{1}{2}} W_{\frac{1}{2}\nu+\frac{1}{4},\, -\frac{1}{4}}(\tfrac{1}{2}z^2)$
	=	$\dfrac{\pi^{\frac{1}{2}}2^{\frac{1}{2}\nu}}{\Gamma(\frac{1}{2}-\frac{1}{2}\nu)}\exp(-\tfrac{1}{4}z^2)\,{}_1F_1(-\tfrac{1}{2}\nu;\ \tfrac{1}{2};\ \tfrac{1}{2}z^2) - \dfrac{\pi^{\frac{1}{2}}2^{\frac{1}{2}\nu+\frac{1}{2}}}{\Gamma(-\frac{1}{2}\nu)} z\exp(-\tfrac{1}{4}z^2)\,{}_1F_1(\tfrac{1}{2}-\tfrac{1}{2}\nu,\ \tfrac{3}{2};\ \tfrac{1}{2}z^2)$
		$D_\nu(z) = \dfrac{\Gamma(\nu+1)}{(2\pi)^{\frac{1}{2}}}\left[i^{\nu}D_{-\nu-1}(iz) + i^{-\nu}D_{-\nu-1}(i^{-1}z)\right]$
		$D_n(z) = \exp(-\tfrac{1}{4}z^2)H_n(z) \qquad D_{-\frac{1}{2}}(z) = (2\pi)^{-\frac{1}{2}}z^{\frac{1}{2}}K_{\frac{1}{4}}(\tfrac{1}{4}z^2)$
		$D_{-1}(z) = (\tfrac{1}{2}\pi)^{\frac{1}{2}}\exp(\tfrac{1}{4}z^2)\operatorname{erfc}(2^{-\frac{1}{2}}z)$
$\mathrm{E}_1(z)$	=	$\int_z^{\infty}\dfrac{e^{-z}}{z}dz$
	=	exponential integral
		$\mathrm{E}_1(i^{\pm 4}z) = \mathrm{E}_1(z) \mp i2\pi$
$\operatorname{erf}(z)$	=	$\dfrac{2}{\sqrt{\pi}}\int_0^z \exp(-z^2)dz = -\operatorname{erf}(-z)$ $\quad \operatorname{erf}(\pm\infty) = \pm 1.$
$\operatorname{erfc}(z)$	=	$\dfrac{2}{\sqrt{\pi}}\int_z^{\infty}\exp(-z^2)dz = 1 - \operatorname{erf}(z)$
f	=	frequency parameter for the cisoidal oscillation $-\infty \leqq f \leqq \infty$ in Parts 1–9. In Part 10 f has real discrete values and in Parts 11–13, f may assume complex values

$f_0 < f < f_1$ = between the indicated limits the coefficient $F(f)$ equals the given function, outside the limits it is zero.

$F(f)$ = coefficient for cisoidal oscillation, parameter f.

$F_n(f)$ = coefficient of an i^n-multiple pair $(F_n(f),\ i^nF_n(g))$ in pairs (223)–(225).

$${}_1F_1(\mu;\ \nu;\ z) = 1 + \frac{\mu}{1!\nu}z + \frac{\mu(\mu+1)}{2!\nu(\nu+1)}z^2 + \cdots$$

= modified Kummer function.

g = epoch; parameter for the unit impulse. $-\infty \leqq g \leqq \infty$ in Parts 1–11. In Parts 12–13, g may assume complex values.

$g_0 < g < g_1$ = between the indicated limits the coefficient $G(g)$ equals the given function, outside the limits it is zero.

$G(g)$ = coefficient for unit impulse, parameter g.

$G(f, g)$ = $\int F(f)\operatorname{cis}(2\pi fg)df$, the indefinite Fourier integral of Part 13.

$$H_n(z) = z^n - \frac{n(n-1)}{2}z^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2\cdot 4}z^{n-4} - \cdots$$

= Hermite polynomial of order n.

$H_\nu^{(1)}(z)$ = Bessel function of the third kind

$$= J_\nu(z) + iY_\nu(z)$$

$$= \frac{2}{\pi}i^{-\nu-1}K_\nu(i^{-1}z) = -i^{-2\nu}H_\nu^{(2)}(i^{-2}z).$$

$$H_\nu^{(1)}(i^{2w}z) = \frac{\sin[(1-w)\nu\pi]}{\sin \nu\pi}H_\nu^{(1)}(z) - i^{-2\nu}\frac{\sin w\nu\pi}{\sin \nu\pi}H_\nu^{(2)}(z).$$

$H_\nu^{(2)}(z)$ = Bessel function of the third kind

$$= J_\nu(z) - iY_\nu(z)$$

$$= \frac{2}{\pi}i^{\nu+1}K_\nu(iz) = -i^{2\nu}H_\nu^{(1)}(i^2z).$$

$$H_\nu^{(2)}(i^{2w}z) = \frac{\sin[(1+w)\nu\pi]}{\sin \nu\pi}H_\nu^{(2)}(z) + i^{2\nu}\frac{\sin w\nu\pi}{\sin \nu\pi}H_\nu^{(1)}(z).$$

$I(z)$ = imaginary part of z. $z = R(z) + iI(z)$.

$I_\nu(z)$ = Bessel function of the first kind for imaginary argument

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2}z)^{\nu+2m}}{m!\,\Gamma(\nu+m+1)}$$

$$= i^{-\nu} J_\nu(iz) = i^{\nu} J_\nu(i^{-1}z)$$

$$I_\nu(i^{2u}z) = i^{2u\nu} I_\nu(z)$$

$j\ k\ l$ = integers greater than zero, infinity not included

$J_\nu(z)$ = Bessel function of the first kind

$$= \sum_{m=0}^{\infty} \frac{(-1)^m(\frac{1}{2}z)^{\nu+2m}}{m!\,\Gamma(\nu+m+1)}$$

$$= i^{\nu} I_\nu(i^{-1}z) = i^{-\nu} I_\nu(iz) = \tfrac{1}{2}[H_\nu^{(1)}(z) + H_\nu^{(2)}(z)]$$

$$J_\nu(i^{2u}z) = i^{2u\nu} J_\nu(z)$$

$K_\nu(z)$ = Bessel function of the second kind for imaginary argument

$$= \frac{\pi}{2 \sin \nu\pi}[I_{-\nu}(z) - I_\nu(z)]$$

$$= \tfrac{1}{2}\pi i^{\nu+1} H_\nu^{(1)}(iz) = \tfrac{1}{2}\pi i^{-\nu-1} H_\nu^{(2)}(i^{-1}z)$$

$$K_\nu(i^{2u}z) = i^{-2u\nu} K_\nu(z) - i\pi \frac{\sin u\nu\pi}{\sin \nu\pi} I_\nu(z)$$

$M_{\mu,\nu}(z)$ $= z^{\frac{1}{2}+\nu} e^{-\frac{1}{2}z} {}_1F_1(\tfrac{1}{2}+\nu-\mu;\ 2\nu+1;\ z)$

$$M_{0,\nu}(z) = 2^{2\nu}\,\Gamma(\nu+1) z^{\frac{1}{2}} I_\nu(\tfrac{1}{2}z)$$

$$M_{\nu-\frac{1}{2},\nu}(z) = 2\nu z^{\frac{1}{2}-\nu} e^{\frac{1}{2}z} \gamma(2\nu, z)$$

$$M_{\mu,\nu}(i^{\pm 2}z) = i^{\pm(2\nu+1)} M_{-\mu,\nu}(z)$$

$$M_{\pm(\nu+\frac{1}{2}),\nu}(z) = z^{\nu+\frac{1}{2}} e^{\mp\frac{1}{2}z} \text{ provided } \nu \neq -\tfrac{1}{2}$$

$m\ n$ = positive integers, including zero, excluding infinity

$\mathfrak{M}(\ )$ = mate of ( )

p = $i2\pi f$, the imaginary radian frequency. In Parts 1–9, unless otherwise stated, the angle of p is taken to be $-\frac{1}{2}\pi$ for negative values of f and $+\frac{1}{2}\pi$ for positive values of f. In Parts 11–13, $p = i2\pi f$ may assume complex values not restricted to the imaginary axis.

$r\ s$ = reals, positive or zero, excluding infinity

$R(z)$ = real part of z, $z = R(z) + iI(z)$

$S(z)$ $= \int_0^z \sin(\frac{1}{2}\pi z^2)dz = -S(-z) \quad S(\pm\infty) = \pm\frac{1}{2}$

$\mathfrak{S}_r(x)$ $= \lim_{a\to\infty} aD_x^r \exp(-\pi a^2x^2)$ = rth singularity function

$\mathfrak{S}_{-k}(x)$ $= \left(\lambda_1 \pm \frac{1}{2(k-1)!}\right)x^{k-1} + \lambda_2 x^{k-2} + \cdots + \lambda_k,\ 0 < \pm x$

t = time, $-\infty \leqq t \leqq \infty$

v, w = integers, positive, negative or zero, but excluding infinity.

$W_{\mu,\nu}(z)$ = confluent hypergeometric function of order μ, ν

$$= \frac{\Gamma(-2\nu)}{\Gamma(\frac{1}{2}-\mu-\nu)} M_{\mu,\nu}(z) + \frac{\Gamma(2\nu)}{\Gamma(\frac{1}{2}-\mu+\nu)} M_{\mu,-\nu}(z)$$

$$W_{0,\nu}(z) = \pi^{-\frac{1}{2}} z^{\frac{1}{2}} K_\nu(\tfrac{1}{2}z).$$

$$W_{\mu,\pm\frac{1}{4}}(z) = 2^{\frac{1}{2}-\mu} z^{\frac{1}{4}} D_{2\mu-\frac{1}{2}}(2^{\frac{1}{2}} z^{\frac{1}{2}}).$$

$$W_{\nu-\frac{1}{2},\pm\nu}(z) = z^{\frac{1}{2}-\nu} e^{\frac{1}{2}z} \Gamma(2\nu, z).$$

$$W_{\nu+\frac{1}{2},\pm\nu}(z) = z^{\nu+\frac{1}{2}} e^{-\frac{1}{2}z}.$$

$$W_{\mu,\nu}(i^{\pm 2}z) = \frac{\Gamma(\mu+\nu+\frac{1}{2})}{\Gamma(\nu-\mu+\frac{1}{2})} i^{\mp(2\nu-1)} W_{-\mu,\nu}(z) + \frac{\Gamma(\mu+\nu+\frac{1}{2})}{\Gamma(2\nu+1)} i^{\pm 2\mu} M_{-\mu,\nu}(z).$$

x, y = reals, unrestricted, but excluding infinity.

Y = admittance of system for cisoidal oscillation.

$Y_\nu(z)$ = Bessel function of the second kind

$$= \frac{\cos \nu\pi \, J_\nu(z) - J_{-\nu}(z)}{\sin \nu\pi}$$

$$= \tfrac{1}{2} i [H_\nu^{(2)}(z) - H_\nu^{(1)}(z)]$$

$$Y_\nu(i^{2w}z) = i^{-2w\nu} Y_\nu(z) + 2i \sin w\nu\pi \operatorname{ctn} \nu\pi \, J_\nu(z).$$

z = complex quantity, not infinite, otherwise unrestricted.

$\bar{z}$ = conjugate of z.

z^μ, br x = $\exp[\mu R(\log z) + i\mu \arg z]$, where $(2x-1)\pi < \arg z \leqq (2x+1)\pi$.

= $e^{i2\pi\mu v} z^\mu$, br$(x-v)$. Branches $(x+v)$, $v = 0, \pm 1, \pm 2, \cdots$ form a complete set and without repetition unless μ is a rational real.

$\alpha, \beta, \gamma, \delta$ = complex quantities, not infinite, real parts greater than zero.

$\Gamma(\nu, z)$ $= \int_z^\infty z^{\nu-1} e^{-z} dz = (\nu-1)\Gamma(\nu-1, z) + z^{\nu-1} e^{-z}.$

$$\Gamma(\nu, 0) = \Gamma(\nu). \quad \Gamma(0, z) = \mathrm{Ei}(z).$$

$$\Gamma(\tfrac{1}{2}, z) = \pi^{\frac{1}{2}} \operatorname{erfc}(z^{\frac{1}{2}}). \quad \Gamma(1, z) = e^{-z}.$$

$$\Gamma(\nu, i^{\pm 4}z) = i^{\pm 4\nu}\Gamma(\nu, z) + \frac{2\pi i^{\pm(2\nu-1)}}{\Gamma(1-\nu)}.$$

$\gamma(\nu, z)$ $= \Gamma(\nu) - \Gamma(\nu, z) = (\nu-1)\gamma(\nu-1, z) - z^{\nu-1} e^{-z}.$

$$\gamma(\tfrac{1}{2}, z) = \pi^{\frac{1}{2}} \operatorname{erf}(z^{\frac{1}{2}}). \quad \gamma(1, z) = 1 - e^{-z}.$$

$$\gamma(\nu, i^{4w}z) = i^{4w\nu}\gamma(\nu, z).$$

$\zeta(z)$ = zeta function of Riemann

$$= \sum_{m=1}^{\infty} \frac{1}{m^z}, \text{ if } 1 < R(z).$$

$\zeta(\nu, z)$ = generalized zeta function

$$= \sum_{m=0}^{\infty} \frac{1}{(z+m)^{\nu}} \quad \text{if } 1 < R(\nu)$$

$$\zeta(\nu, 1) = \zeta(\nu) \qquad \zeta(2, z) = \psi'(z)$$

$$\zeta(k+1, z) = \frac{(-1)^{k+1}}{k!}\psi^{(k)}(z)$$

θ = real quantity, unrestricted but excluding infinity

λ, μ, ν = complex quantities, not infinite, otherwise unrestricted

ρ, σ, τ = complex quantities, not infinite, real parts not less than zero

$\phi_n(x)$ = $\exp(\pi x^2) D_x^{\,n} \exp(-2\pi x^2)$

= $(-2\pi^{\frac{1}{2}})^n D_n(2\pi^{\frac{1}{2}} x)$ where D_n is the parabolic cylinder function of order n

= $(-2\pi^{\frac{1}{2}})^n \exp(-\pi x^2) H_n(2\pi^{\frac{1}{2}} x)$ where H_n is the Hermite polynomial of order n

$\chi_n(x)$ = $\exp(-\pi x^2)\, {}_1F_1(-n, \tfrac{3}{2} - \tfrac{1}{2}n, \pi x^2)$

$\psi(z)$ = $\Gamma'(z)/\Gamma(z)$ = logarithmic derivative of the gamma function; $-\psi(1)$ = Euler's constant = 0.5772

$\boxed{A}$ = alternative expression for this pair

$\boxed{B}$ = border case of this pair lying outside the domain defined by the standardized notation for parameters. Reference must be made to this case at its place in the table for the complete definition of the domain

$\boxed{C}$ = continuation of the domain defined by the standardized notation for parameters. All conditions to the left of the semicolon, if any, must be satisfied if this pair is to be valid for the extensions of the parameters at the right

$\boxed{D}$ = decrease of the domain defined by the standardized notation for parameters. This pair is not valid if the parameters satisfy all the conditions which follow

$\boxed{F}$ = further information which is needed for the complete statement of this pair

$\boxed{L}$ = limit pair derived from this regular pair by allowing one or more parameters to pass to a limit

$\boxed{M}$ = multiple valued function restricted here to the value indicated

$\boxed{N}$ = notation used here to simplify the writing of this particular pair

P = path of integration, whenever it differs from the entire real axis of f. The particular paths are specified in terms of $p = i2\pi f$, although the integration is always with respect to f.

P $x - i|\infty|$ to $x + i|\infty|$; $R(\lambda) < x$, $R(\mu) < x$, $\cdots$ = straight line path in the p plane parallel to the imaginary axis, crossing the real axis at the point $p = x$, so chosen that the points λ, μ, $\cdots$ lie on the left of the path.

P $(-|\infty| \operatorname{cis} \theta; \lambda +, \mu -, \cdots); \nu, \cdots$ = continuous path in the p plane starting from infinity asymptotically to a line through the origin making an angle θ with the negative real axis, encircling the points λ, μ, $\cdots$ in the positive or negative direction as indicated, and returning to infinity asymptotically to the same line. The points ν, $\cdots$ are outside this closed path. For a closed finite continuous path, the reference to infinity is omitted.

R = restricted case of this pair requiring restrictions of the domain defined by the standardized notation for parameters. Reference must be made to this case at its place in the table for the complete definition of the domain.

S = special case of this pair within the domain defined by the standardized notation for parameters.

SUMMARY OF PARAMETER NOTATION

	Not Infinite, Otherwise Unrestricted	Not Infinite, and Real Part $\geqq 0$	Not Infinite, and Real Part > 0
Integers	v, w	m, n	j, k, l
Reals	x, y, θ	r, s	a, b, c
Complex	z, λ, μ, ν	ρ, σ, τ	$\alpha, \beta, \gamma, \delta$

Explanation of Table I

(Condensed from the text)

This table omits the integral s gn and lists merely the functions $F(f)$ and $G(g)$ which are referred to either as paired coefficients or as a pair The integral heading Table I may be restored with respect to either coefficient. The 649 pairs in Parts 4–9 of the table are arranged according to the function $F(f)$ rational functions coming first and transcendental functions last The substitution $p = i2\pi f$ is used If $g_0 < g < g_1$ occurs it signifies that between the indicated limits $G(g)$ equals the given function outside these limits it is zero Similarly for $f_0 < f < f_1$ and $F(f)$ Certain pairs show the $F(f)$ coefficients as the limits of more general functions these pairs are obtained not by d rect Riemann integration but by such integration followed by passage to a limit Without further information than this the simpler applications of the table may be made

In most cases the integral formulas are valid for integration in the sense of Riemann only for lim ted regions of values of the various parameters involved these regions are indicated by the choice of letters used exceptions being noted by squared letters listed under each pair For complete definitions of the domains of the parameters and the significance of the squared letters as well as for the functions employed see the preceding section on Notation The squared letters are summarized as follows

$\boxed{A}$ = alternative expression	$\boxed{M}$ = multiple-valued
$\boxed{B}$ = border case	$\boxed{N}$ = notation
$\boxed{C}$ = continuation of doma n	$\boxed{P}$ = path of integration
$\boxed{D}$ = decrease of domain	$\boxed{R}$ = restricted domain case,
$\boxed{F}$ = further information	$\boxed{S}$ = special case
$\boxed{L}$ = l mit case	

Additional pairs may be derived by using the elementary transformations and combinations of Part 2

Parts 10–13 extend the table to include a few Fourier series and contour integrals which are explained under the headings of the parts and in the text.

TABLE I—A TABLE OF FOURIER INTEGRALS*

$$\int_{-\infty}^{\infty} F(f)e^{i2\pi fg}df = G(g)$$

$$\int_{-\infty}^{\infty} G(g)e^{-i2\pi fg}dg = F(f)$$

No.	Coefficient $F(f)$	Coefficient $G(g)$
	Part 1. General Processes for Deriving the Mate	
101	$F(f)$	$G(g) = \int_{-\infty}^{\infty} F(f)\text{cis}(2\pi fg)df$
102	$F(f) = \int_{-\infty}^{\infty} G(g)\text{cis}(-2\pi fg)dg$	$G(g)$
103	$F(f)$	$D_g \int_{-\infty}^{\infty} F(f)\text{cis}(2\pi fg)p^{-1}df$
104	$\lambda_1(p-p_0)+\lambda_2(p-p_0)^2+\cdots$ $\boxed{\text{E}}$ using pairs 401.1 and 206	$\text{cis}\,(2\pi f_0 g)[\lambda_1\mathfrak{S}_1(g)+\lambda_2\mathfrak{S}_2(g)+\cdots]$
105	$\lambda_1\frac{1}{(p-p_0)}+\lambda_2\frac{1}{(p-p_0)^2}+\cdots$ $\boxed{\text{E}}$ using pairs 408.1 and 206	$\text{cis}\,(2\pi f_0 g)\left(\lambda_1+\lambda_2\frac{g}{1!}+\lambda_3\frac{g^2}{2!}+\lambda_4\frac{g^3}{3!}+\cdots\right),\quad 0<g$
106	$\lambda_1 p+\lambda_2 p^2+\lambda_3 p^3+\cdots$ $\boxed{\text{E}}$ using pair 401.1	$\lambda_1\mathfrak{S}_1(g)+\lambda_2\mathfrak{S}_2(g)+\lambda_3\mathfrak{S}_3(g)+\cdots$

* Every pair in Parts 1–9 of Table I gives

(1) the evaluated Fourier integrals,

$$\int_{-\infty}^{\infty} F(f)\,\text{cis}\,(2\pi fg)df = G(g), \qquad \int_{-\infty}^{\infty} G(g)\,\text{cis}\,(2\pi fg)dg = F(-f),$$

$$\int_{-\infty}^{\infty} F(f)\cos(2\pi fg)df = \tfrac{1}{2}[G(g)+G(-g)], \qquad \int_{-\infty}^{\infty} G(g)\cos(2\pi fg)dg = \tfrac{1}{2}[F(f)+F(-f)],$$

$$\int_{-\infty}^{\infty} F(f)\sin(2\pi fg)df = -i\tfrac{1}{2}[G(g)-G(-g)], \qquad \int_{-\infty}^{\infty} G(g)\sin(2\pi fg)dg = i\tfrac{1}{2}[F(f)-F(-f)],$$

(2) the explicit expression in g of the result of the operation $F[p/(i2\pi)]$ applied to the unit impulse function $\mathfrak{S}_0(g)$, where the operator $p = i2\pi f = d/dg$.

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
107	$\lambda_1 \frac{1}{p} + \lambda_2 \frac{1}{p^2} + \lambda_3 \frac{1}{p^3} + \cdots$ E using pair 408 1	$\lambda_1 + \lambda_2 \frac{g}{1!} + \lambda_3 \frac{g^2}{2!} + \lambda_4 \frac{g^3}{3!} + \cdots, \quad 0 < g$
08	$\frac{1}{p^a}\left(\lambda_0 + \lambda_1 \frac{1}{p} + \lambda_2 \frac{1}{p^2} + \cdots\right)$ C $1 \leqq a$ E using pair 516	$\frac{1}{\Gamma(a) g^{1-a}}\left[\lambda_0 + \lambda_1 \frac{1}{a} g + \lambda_2 \frac{1}{a(a+1)} g^2 + \cdots\right], \quad 0 < g$
09	$\frac{1}{p^a}(\lambda_0 + \lambda_1 p + \lambda_2 p^2 + \cdots)$ C $1 \leqq a$ E using pair 501	$\frac{1}{\Gamma(a) g^{1-a}}\left[\lambda_0 - \lambda_1 (1-a) \frac{1}{g} + \lambda_2 (1-a)(2-a) \frac{1}{g^2} + \cdots\right], \quad 0 < g$
10	$\frac{1}{\sqrt{p}}\left(\lambda_0 + \lambda_1 \frac{1}{p} + \lambda_2 \frac{1}{p^2} + \lambda_3 \frac{1}{p^3} + \cdots\right)$ E using pair 518	$\frac{1}{\sqrt{\pi g}}\left[\lambda_0 + \lambda_1 \frac{2g}{1} + \lambda_2 \frac{(2g)^2}{1 \cdot 3} + \lambda_3 \frac{(2g)^3}{1 \cdot 3 \cdot 5} + \cdots\right], \quad 0 < g$
11	$\frac{1}{\sqrt{p}}(\lambda_0 + \lambda_1 p + \lambda_2 p^2 + \lambda_3 p^3 + \cdots)$ E using pair 502 1	$\frac{1}{\sqrt{\pi g}}\left[\lambda_0 - \lambda_1 \frac{1}{2g} + \lambda_2 \frac{1 \cdot 3}{(2g)^2} - \lambda_3 \frac{1 \cdot 3 \cdot 5}{(2g)^3} + \cdots\right], \quad 0 < g$
12	$\lambda_0 \phi_0(f) + \lambda_1 \phi_1(f) + \lambda_2 \phi_2(f) + \cdots$	$\lambda_0 \phi_0(g) + i\lambda_1 \phi_1(g) + i^2 \lambda_2 \phi_2(g) + \cdots$
13 1	$F(f) = 0$, except at $f = v/(2\pi)$, $v = 0, \pm 1, \pm 2, \ldots$, where it becomes infinite covering at the limit the area $A_v \lambda^v = \frac{1}{2\pi}\int_{-\pi}^{\pi} G(g) e^{-ivg} dg$ R $z = \lambda e^{ig}$ H A_v may be complex	$G(g) = G\left(i \log \frac{\lambda}{z}\right)$ $= \sum_{v=-\infty}^{\infty} A_v z^v$ $= \sum_{v=-\infty}^{\infty} A_v \lambda^v e^{ivg}$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
	Part 2. Elementary Combinations and Transformations	
201	$F_1 \pm F_2$	$G_1 \pm G_2$
202 †	$F_1 F_2$	$\int_{-\infty}^{\infty} G_1(x) G_2(g - x) dx$
203	$\int_{-\infty}^{\infty} F_1(-x) F_2(f + x) dx$	$G_1 G_2$
204	λF	λG
205	$F(af)$	$\frac{1}{a} G\left(\frac{g}{a}\right)$
206	$F(f - f_0) = F\left(\frac{p - p_0}{i2\pi}\right)$	$\mathrm{cis}(2\pi f_0 g) G = e^{p_0 g} G$
207	$\mathrm{cis}(-2\pi f g_0) F = e^{-p g_0} F$	$G(g - g_0)$
208	pF	$D_g G$
209	$D_p F = \frac{1}{i2\pi} D_f F$	$-gG$
210	$\frac{1}{p} F$	$\int_{-\infty}^{g} G dg = D_g^{-1} G$
211	$\int_{-i\infty}^{p} F dp = i2\pi \int_{-\infty}^{f} F df = D_p^{-1} F$	$-\frac{1}{g} G$

† From (202) or (203), with g (or f) $= 0$, and (215) and (217) follow the important identities for the integrated product of two pairs of coefficients and for the integrated squared moduli of a pair of coefficients:

$$\int_{-\infty}^{\infty} F_1(f) F_2(\pm f) df = \int_{-\infty}^{\infty} G_1(g) G_2(\mp g) dg,$$

$$\int_{-\infty}^{\infty} |F^2| df = \int_{-\infty}^{\infty} |G^2| dg,$$

$$\int_{-\infty}^{\infty} F_1(x) G_2(x) dx = \int_{-\infty}^{\infty} G_1(x) F_2(x) dx.$$

The symmetry of these identities is to be noted; this would not be the case if the radian frequency $2\pi f$ were employed in place of the cyclic frequency f.

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
212	$D_\lambda F$	$D_\lambda G$
213	$\int_{\lambda_0}^{\lambda} F d\lambda = D_\lambda^{-1} F$	$\int_{\lambda_0}^{\lambda} G d\lambda = D_\lambda^{-1} G$
214	$F(-f)$	$G(-g)$
214.1	$F(f) \pm F(-f)$, even and odd partition	$G(g) \pm G(-g)$
215	$\bar{F}(\pm f)$	$\bar{G}(\mp g)$
216	$F(f) \pm \bar{F}(f)$	$G(g) \pm \bar{G}(-g)$
217	$G(\pm f)$	$F(\mp g)$
218	$G(\pm ip)$	$\frac{1}{2\pi} F\left(\frac{\pm g}{2\pi}\right)$
219	$\frac{F(f)}{p - p_0}$	$e^{p_0 g}\int_{-\infty}^{g} e^{-p_0 g} G(g) dg$
220	$\frac{p}{p - p_0} F(f)$	$G(g) + p_0 e^{p_0 g}\int_{-\infty}^{g} e^{-p_0 g} G(g) dg$
221	$f^{-\frac{1}{2}} D_f^{n}(f^{n+\frac{1}{2}} F) = (\frac{1}{2} + f D_f)^{n} F$	$-g^{-\frac{1}{2}} D_g^{n}(g^{n+\frac{1}{2}} G) = -(\frac{1}{2} + g D_g)^{n} G$
222	$e^{\pm\pi f^2} D_f^{n}(e^{\mp\pi f^2} F) = (\mp 2\pi f + D_f)^{n} F$	$i^{\pm n} e^{\pm\pi g^2} D_g^{n}(e^{\mp\pi g^2} G) = i^{\pm n}(\mp 2\pi g + D_g)^{n} G$
223‡	$F_n(f)$ $n = 0, 1, 2, \cdots, 11$ $F_n(f) = \frac{1}{4}[F(f) + i^{3n} F(-f) + i^{-n} G(f) + i^{n} G(-f)]$ $F_{n+4} = R(F_n)$, $F_{n+8} = I(F_n)$, if $n = 0, 1, 2, 3$	$i^{n} F_n(g)$

‡ The coefficients of the i^n multiple pairs satisfy the following integral equations

$$F_n(f) = (-1)^{\frac{1}{2}n}\, 2\int_0^\infty F_n(g)\cos(2\pi f g)\,dg \qquad n = 0, 2, 4, 6, 8, 10$$

$$F_n(f) = (-1)^{\frac{1}{2}(n-1)}\, 2\int_0^\infty F_n(g)\sin(2\pi f g)\,dg, \qquad n = 1, 3, 5, 7, 9, 11$$

No.	Coefficient $F(f)$	Coefficient $G(g)$
224	$F_0(f) + F_1(f) + F_2(f) + F_3(f)$ [N] F_n is as for pair 223	$F_0(g) + iF_1(g) - F_2(g) - iF_3(g)$
225	$F_4(f) + F_6(f) + F_5(f) + F_7(f)$ $+ i[F_8(f) + F_{10}(f) + F_9(f) + F_{11}(f)]$ [N] F_n is as for pair 223	$F_4(g) - F_6(g) - F_9(g) + F_{11}(g)$ $+ i[F_8(g) - F_{10}(g) + F_5(g) - F_7(g)]$.
226.1	$\cdots, B_v A_v \lambda^v, \cdots$ [N] $B_{v+k} = B_v$ [N] $e^{i2\pi/k} = \mu$ [N] A_v and B_v may be complex	$\dfrac{1}{k}\sum_{v=0}^{k-1} B_v \sum_{m=0}^{k-1} \mu^{-mv} G(\mu^m g)$

Part 3. Key Pairs

These key pairs have been brought together from Parts 4–9 of Table I, omitting however the explanatory notes referring to special cases, extensions and restrictions of the domain, etc. These notes may be found by reference to the key pairs in their proper numerical order in Parts 4–9 of the table.

301.1 Key	$\dfrac{1}{\cos^\alpha[\gamma(p+\lambda)]}$ [A] pair 607.0	$\dfrac{2^{\alpha-2}e^{-\lambda g}}{\pi\gamma\Gamma(\alpha)}\Gamma\left(\dfrac{\alpha\gamma+ig}{2\gamma}\right)\Gamma\left(\dfrac{\alpha\gamma-ig}{2\gamma}\right)$
302.1 Key	$\dfrac{1}{p+\lambda}\left\{\dfrac{1}{\sin[\gamma(p+\lambda)]} - \dfrac{1}{\gamma(p+\lambda)}\right\}$ [A] pair 608.0	$\dfrac{1}{\pi}e^{-\lambda g}\log\left[1+\exp\left(-\dfrac{\pi}{\gamma}\lvert g\rvert\right)\right]$
303.1 Key	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}}[\lambda+(p+\rho)^{\frac{1}{2}}]}$ [A] pair 830.0	$\exp(\lambda\sigma^{\frac{1}{2}} + \lambda^2 g - \rho g)\,\mathrm{erfc}\left(\lambda g^{\frac{1}{2}} + \dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right),$ $0 < g$
304.1 Key	$\dfrac{\exp\{\mu[(p+\rho)^{\frac{1}{2}} - (p+\sigma)^{\frac{1}{2}}]^2\}}{(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}[(p+\rho)^{\frac{1}{2}}+(p+\sigma)^{\frac{1}{2}}]^{2\alpha-2}}$ [A] pair 870.1	$\dfrac{g^{\frac{1}{2}\alpha-\frac{1}{2}}e^{-\frac{1}{2}(\rho+\sigma)g}I_{\alpha-1}[\frac{1}{2}(\rho-\sigma)g^{\frac{1}{2}}(g+4\mu)^{\frac{1}{2}}]}{(\rho-\sigma)^{\alpha-1}(g+4\mu)^{\frac{1}{2}\alpha-\frac{1}{2}}},$ $0 < g$

TABLE 1 (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
305 1 Key	$\exp(\sigma^2 p)K_\nu[\sigma^2(p+\rho)]$ ⊠ pair 909 1	$\frac{\exp[-\rho(g+\sigma^2)]}{g^{\frac12}(g+2\sigma^2)^{\frac12}}\cosh\left[\nu\cosh^{-1}\left(\frac{g+\sigma^2}{\sigma^2}\right)\right]$ $=\frac{\exp[-\rho(g+\sigma^2)]}{2g^{\frac12}(g+2\sigma^2)^{\frac12}}\left\{\left[\frac{g^{\frac12}+(g+2\sigma^2)^{\frac12}}{2^{\frac12}\sigma}\right]^{2\nu}\right.$ $\left.+\left[\frac{g^{\frac12}+(g+2\sigma^2)^{\frac12}}{2^{\frac12}\sigma}\right]^{-2\nu}\right\}\quad 0<g$
306 1 Key	$p^{\frac12}J_{-\frac12}(\alpha p^{\frac12})$ ⊠ pair 923 8	$\frac{g^{\frac12}}{2^{\frac12}\pi^{\frac12}\alpha}J_{-\frac12}\left(\frac{g^2}{4\alpha}\right)$
307 1 Key	$\frac{\exp(\tau^2 p)K_{\nu-\frac12}[\tau^2(p+\rho)^{\frac12}(p+\sigma)^{\frac12}]}{(p+\rho)^{\frac12\nu-\frac14}(p+\sigma)^{\frac12\nu-\frac14}}$ ⊠ pair 929 1	$\frac{2^{\nu-\frac12}\pi^{\frac12}}{\tau^{2\nu-1}(\rho-\sigma)^{\nu-\frac12}}g^{\frac12(\nu-1)}(g+2\tau^2)^{\frac12\nu-\frac12}$ $\times\exp[-\frac12(\rho+\sigma)(g+\tau^2)]$ $\times I_{\nu-\frac12}[\frac12(\rho-\sigma)g^{\frac12}(g+2\tau^2)^{\frac12}]\quad 0<g$
308 1 Key	$(p+\rho)^{\frac12}\{J_{\nu+\frac12}[\sigma(p+\rho)]Y_{\nu-\frac12}[\sigma(p+\rho)]$ $-J_{\nu-\frac12}[\sigma(p+\rho)]Y_{\nu+\frac12}[\sigma(p+\rho)]\}$ ⊠ pair 938 0	$\frac{(2\sigma)^{2\nu}e^{-\rho g}[g+(g^2+4\sigma^2)^{\frac12}]^{-2\nu}}{(\frac12\pi)^{\frac12}g^{\frac12}(g^2+4\sigma^2)^{\frac12}},\quad 0<g$
309 1 Key	$\exp(4\tau^2 p)I_{\nu-\frac12}\{\tau^2[(p+\rho)^{\frac12}-(p+\sigma)^{\frac12}]^2\}$ $\times K_{\nu-\frac12}\{\tau^2[(p+\rho)^{\frac12}+(p+\sigma)^{\frac12}]^2\}$ ⊠ pair 940 1	$\frac{\exp[-\frac12(\rho+\sigma)(g+4\tau^2)]}{g^{\frac12}(g+8\tau^2)^{\frac12}}$ $\times I_{2\nu-1}[\frac12(\rho-\sigma)g^{\frac12}(g+8\tau^2)^{\frac12}]\quad 0<g$
310 1 Key	$K_\nu[(\sigma^{\frac12}+\tau^{\frac12})(p+\rho)^{\frac12}]$ $\times I_\nu[(\sigma^{\frac12}-\tau^{\frac12})(p+\rho)^{\frac12}]$ ⊠ pair 945 1	$\frac{1}{2g}\exp\left(-\rho g-\frac{\sigma+\tau}{2g}\right)I_\nu\left(\frac{\sigma-\tau}{2g}\right),$ $0<g$
311 1 Key	$\frac{1}{(p+\rho)}\exp(\frac12\sigma^2 p)W_{\nu-\alpha,\nu-\frac12}[\sigma^2(p+\rho)]$ ⊠ pair 961 1	$\frac{\exp[-\rho(g+\frac12\sigma^2)]}{\sigma^{2\nu-1}\Gamma(\alpha)}g^{\alpha-1}(g+\sigma^2)^{2\nu-\alpha-1},$ $0<g$
312 1 Key	$\frac{1}{(p+\rho)^{\alpha-\nu}}\exp\left[\frac{1}{2\lambda(p+\rho)}\right]$ $\times M_{\nu-\alpha,\nu-\frac12}\left[\frac{1}{\lambda(p+\rho)}\right]$ ⊠ pair 963 1	$\frac{\Gamma(2\nu)}{\lambda^{\nu}\Gamma(\alpha)}e^{-\rho g}g^{\alpha-\nu-1}I_{2\nu-1}\left(\frac{2g^{\frac12}}{\lambda^{\frac12}}\right),\quad 0<g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
313.1 Key	$\frac{1}{(p+\rho)^{2\nu}}\exp\left[\frac{1}{2}\sigma(p+\rho)^2\right]$ $\times W_{\nu-\alpha,\nu-\frac{1}{2}}[\sigma(p+\rho)^2]$ $\boxed{A}$ pair 965.1	$\frac{2^{\alpha+\frac{1}{2}}\sigma^{\nu-\frac{1}{2}\alpha+\frac{1}{4}}}{\Gamma(2\alpha)}g^{\alpha-\frac{3}{2}}\exp\left(-\rho g-\frac{g^2}{8\sigma}\right)$ $\times M_{\frac{1}{4}-2\nu+\frac{1}{2}\alpha,\frac{1}{2}\alpha-\frac{1}{4}}\left(\frac{g^2}{4\sigma}\right),\quad 0<g$
314.1 Key	$\zeta[\alpha+1,\gamma(p+\beta)]$ $\boxed{A}$ pair 971.1	$\frac{g^{\alpha}e^{-\beta g}}{\gamma^{\alpha+1}\Gamma(\alpha+1)\left[1-\exp\left(-\frac{g}{\gamma}\right)\right]},\quad 0<g$
315.1 Key	$\Gamma(p+\beta+1)\zeta(p+\beta+1)$ $\boxed{A}$ pair 973.1	$\frac{e^{-(\beta+1)g}}{\exp(e^{-g})-1}$

Part 4. Rational Algebraic Functions of f

No.	Coefficient $F(f)$	Coefficient $G(g)$
401.1	$p^n=\lim\limits_{a\to 0}\left[p^n\exp(-\pi a^2f^2)\right]$	$\mathfrak{S}_n(g)=\lim\limits_{a\to 0}\left[\frac{1}{a}D_g{}^n\exp\left(-\frac{\pi g^2}{a^2}\right)\right]$
402	$p^n=\lim\limits_{a\to 0}\sum\limits_{k=1}^{n+1}\frac{(-1)^k(n+1)!(e^{-kap}-1)}{k!(n-k+1)!a^{n+1}p}$	$\mathfrak{S}_n(g)$
403.1	$1=\lim\limits_{\beta\to 0}\left[\exp(-\beta\|p-p_0\|)\right]$	$\mathfrak{S}_0(g)$, unit impulse at $g=0$
404.1	$p=\lim\limits_{\beta\to 0}\left[p\exp(-\pi\beta f^2)\right]$	$\mathfrak{S}_1(g)$, negative unit doublet at $g=0$
405	$\|p\|^{2n}=\lim\limits_{\beta\to 0}(\|p\|^{2n}e^{-\beta\|p\|})$	$(-1)^n\mathfrak{S}_{2n}(g)$
406	$\|p\|^{2n+1}=\lim\limits_{\beta\to 0}(\|p\|^{2n+1}e^{-\beta\|p\|})$ $\boxed{S}$ $n=0$: pair 407	$\frac{(-1)^{n+1}(2n+1)!}{\pi g^{2n+2}}$
407	$\|p\|=\lim\limits_{\beta\to 0}(\|p\|e^{-\beta\|p\|})$	$-\frac{1}{\pi g^2}$
408.1	$\frac{1}{p^k}=\lim\limits_{\beta\to 0}\left[\frac{1}{(p\pm\beta)^k}\right]$	$\frac{\pm g^{k-1}}{(k-1)!},\quad 0<\pm g$

No	Coefficient $F(f)$	Coefficient $G(g)$
410	$\dfrac{1}{p^k} = \lim\limits_{\beta\to 0}\left[\dfrac{1}{p^k}\exp(-\pi\beta f^2)\right]$ $\boxed{F}$ This pair is given as the limit approached by the formal pair 726 2 not as the limit approached by a regular pair	$\mathfrak{P}_{-k}(g) = \lim\limits_{\beta\to 0}\left[\dfrac{1}{\beta^{\frac{1}{2}}} D_g^{-k}\exp\left(-\dfrac{\pi g^2}{\beta}\right)\right]$
411 1	$\dfrac{1}{p^k} = \lim\limits_{\beta\to 0}\left\{\dfrac{\frac{1}{2}}{(p-\beta)^k} + \dfrac{\frac{1}{2}}{(p+\beta)^k} + \sum_{j=1}^{k}\left[\dfrac{\lambda_j (k-j)!}{(p+\beta)^{k-j+1}} - \dfrac{\lambda_j (k-j)!}{(p-\beta)^{k-j+1}}\right]\right\}$ $\boxed{S}$ $k = 1\ 2$ pairs 415 416	$\mathfrak{P}_{-k}(g) = \left[\lambda_1 \pm \dfrac{1}{2(k-1)!}\right]g^{k-1} + \lambda_2 g^{k-2} + \lambda_3 g^{k-3} + \quad + \lambda_k,$ $\quad 0 < \pm g$
415	$\dfrac{1}{p} = \lim\limits_{\beta\to 0}\left(\dfrac{\frac{1}{2}-\lambda}{p-\beta} + \dfrac{\frac{1}{2}+\lambda}{p+\beta}\right)$	$\mathfrak{P}_{-1}(g) = \lambda \pm \frac{1}{2},$ $\quad 0 < \pm g$ unit step at $g = 0$
416	$\dfrac{1}{p^2} = \lim\limits_{\beta\to 0}\left[\dfrac{\frac{1}{2}-\lambda}{(p-\beta)^2} + \dfrac{\frac{1}{2}+\lambda}{(p+\beta)^2} - \dfrac{2\beta\mu}{p^2-\beta^2}\right]$	$\mathfrak{P}_{-2}(g) = \frac{1}{2}\|g\| + \lambda g + \mu$
421	$F(f) = \lim\limits_{a\to 0}\Phi(f)$ $\boxed{H}$ $\Phi(f)$ is any proper rational fraction in p with n distinct poles the order of pole s_j being n_j All pure imaginary poles in F are the limits of corresponding poles in Φ which have assigned real parts $\mp a$ $\boxed{F}$ This is a regular pair if there are no pure imaginary poles in F	$\sum_{j=1}^{n}\sum_{k=1}^{n_j} \pm \lambda_{jk} e^{s_j g} g^{n_j-k},$ $\quad 0 < \pm g$ $\boxed{H}$ $\lambda_{jk} = \dfrac{\{D_p^{k-1}[F(f)(p-s_j)^{n_j}]\}_{p=s_j}}{(k-1)!(n_j-k)!}$ $\boxed{H}$ The upper or lower signs for each term are employed according as the real part of s_j (either actual or vestigial) is less than or greater than zero
431	$\dfrac{1}{(p\pm\beta)^k}$ $\boxed{S}$ $k = 1, 2, 3$ pairs 438 and 439, 442 450 $\boxed{L}$ $\beta \to 0$ pair 408 1	$\dfrac{\pm 1}{(k-1)!} g^{k-1} e^{\mp\beta g},$ $\quad 0 < \pm g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
433	$\dfrac{1}{(p^2-\beta^2)^k}$ $\boxed{S}$ $k = 1$: pair 444	$\dfrac{(-1)^k}{(k-1)!} e^{-\beta\|g\|} \sum_{j=1}^{k} \dfrac{(k+j-2)!\,\|g\|^{k-j}}{(j-1)!\,(k-j)!\,(2\beta)^{k+j-1}}$ $= \dfrac{(-1)^k \|g\|^{k-\frac{1}{2}} K_{k-\frac{1}{2}}(\beta\|g\|)}{(k-1)!\,\pi^{\frac{1}{2}}(2\beta)^{k-\frac{1}{2}}}$
438	$\dfrac{1}{p+\beta}$	$e^{-\beta g}$, $\qquad 0 < g$
439	$\dfrac{1}{p-\beta}$	$-e^{\beta g}$, $\qquad g < 0$
440	$\dfrac{1}{p-p_0} = \lim_{\beta\to 0}\left(\dfrac{\frac{1}{2}-\lambda}{p-p_0-\beta} + \dfrac{\frac{1}{2}+\lambda}{p-p_0+\beta}\right)$ $\boxed{S}$ $p_0 = 0$: pair 415	$\operatorname{cis}(2\pi f_0 g)\mathfrak{S}_{-1}(g)$
442	$\dfrac{1}{(p\pm\beta)^2}$	$\pm g e^{\mp\beta g}$, $\qquad 0 < \pm g$
444	$\dfrac{1}{p^2-\beta^2}$	$-\dfrac{1}{2\beta} e^{-\beta\|g\|}$
445	$\dfrac{p}{p^2-\beta^2}$	$\pm \frac{1}{2} e^{-\beta\|g\|}$, $\qquad 0 < \pm g$
446.1	$\dfrac{1}{p^2+x^2} = \dfrac{1}{2ix}\lim_{\beta\to 0}\left(\dfrac{\frac{1}{2}-\lambda}{p-ix-\beta} + \dfrac{\frac{1}{2}+\lambda}{p-ix+\beta} - \dfrac{\frac{1}{2}-\lambda}{p+ix-\beta} - \dfrac{\frac{1}{2}+\lambda}{p+ix+\beta}\right)$ $\boxed{O}$ $x = 0$	$\dfrac{1}{x}\sin xg\, \mathfrak{S}_{-1}(g)$
447.1	$\dfrac{p}{p^2+x^2} = \frac{1}{2}\lim_{\beta\to 0}\left(\dfrac{\frac{1}{2}-\lambda}{p-ix-\beta} + \dfrac{\frac{1}{2}+\lambda}{p-ix+\beta} + \dfrac{\frac{1}{2}-\lambda}{p+ix-\beta} + \dfrac{\frac{1}{2}+\lambda}{p+ix+\beta}\right)$ $\boxed{S}$ $x = 0$: pair 415	$\cos xg\, \mathfrak{S}_{-1}(g)$

No	Coefficient $F(f)$	Coefficient $G(g)$	
448	$\dfrac{1}{(p \pm \alpha)(p \pm \beta)}$ pair 448.1 $\alpha = \beta$ pair 442 $\alpha = \infty$ or $\beta = \infty$ pairs 438 and 439	$\dfrac{e^{\mp \beta g} - e^{\mp \alpha g}}{\alpha - \beta}$	$0 < \pm g$
448.1	$\dfrac{1}{(p \pm \beta)^2 + \lambda^2}$ pair 448 $\lambda = 0$ pair 442 $\beta = \infty$ and $\lambda = \infty$ pairs 438 and 439 $R(\beta) \leqq \lvert I(\lambda) \rvert$	$\pm \dfrac{1}{\lambda} \sin \lambda g \, e^{\mp \beta g}$	$0 < \pm g$
448.8	$\dfrac{1}{(p + \alpha)(p - \beta)}$ $\alpha = \beta$ pair 444 $\alpha = \infty$ pair 439 $\beta = \infty$ pair 438	$\dfrac{-1}{\alpha + \beta} e^{-\alpha g}$	$0 < g$
		$\dfrac{-1}{\alpha + \beta} e^{\beta g}$	$g < 0$
449	$\dfrac{p}{(p \pm \alpha)(p \pm \beta)}$ pair 449.1 $\alpha = \beta$ pair 449.5 $\alpha = 0$ or $\beta = 0$ pairs 438 and 439	$\dfrac{\pm (\alpha e^{\mp \alpha g} - \beta e^{\mp \beta g})}{\alpha - \beta}$	$0 < \pm g$
449.1	$\dfrac{p}{(p \pm \beta)^2 + \lambda^2}$ pair 449 $\lambda = 0$ pair 449.5 $\beta^2 + \lambda^2 = 0$ pairs 438 and 439 $R(\beta) \leqq \lvert I(\lambda) \rvert$	$\left(\pm \cos \lambda g - \dfrac{\beta}{\lambda} \sin \lambda g \right) e^{\mp \beta g}$	$0 < \pm g$
449.5	$\dfrac{p}{(p \pm \beta)^2}$	$(\pm 1 - \beta g) e^{\mp \beta g}$	$0 < \pm g$
449.8	$\dfrac{p}{(p + \alpha)(p - \beta)}$ $\alpha = \beta$ pair 445 $\alpha = 0$ pair 439 $\beta = 0$ pair 438	$\dfrac{\alpha}{\alpha + \beta} e^{-\alpha g}$	$0 < g$
		$-\dfrac{\beta}{\alpha + \beta} e^{\beta g}$	$g < 0$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
450	$\dfrac{1}{(p \pm \beta)^3}$	$\pm \frac{1}{2} g^2 e^{\mp \beta g}, \quad 0 < \pm g$
452	$\dfrac{1}{(p+\alpha)(p+\beta)(p+\gamma)}$ $\boxed{S}$ $\alpha=\beta$ or $\beta=\gamma$ or $\gamma=\alpha$: pair 452.5 $\boxed{S}$ $\alpha+\beta=2\gamma$ or $\beta+\gamma=2\alpha$ or $\gamma+\alpha=2\beta$: pair 460.3 $\boxed{B}$ $\alpha=\infty$ or $\beta=\infty$ or $\gamma=\infty$: pair 448	$\dfrac{(\gamma-\beta)e^{-\alpha g}+(\alpha-\gamma)e^{-\beta g}+(\beta-\alpha)e^{-\gamma g}}{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}, \quad 0 < g$
452.5	$\dfrac{1}{(p+\alpha)(p+\beta)^2}$ $\boxed{S}$ $\alpha = \beta$: pair 450 $\boxed{B}$ $\alpha = \infty$: pair 442 $\boxed{B}$ $\beta = \infty$: pair 438	$\dfrac{e^{-\alpha g} - e^{-\beta g}[1 - (\alpha - \beta)g]}{(\alpha - \beta)^2}, \quad 0 < g$
453	$\dfrac{p}{(p+\alpha)(p+\beta)(p+\gamma)}$ $\boxed{S}$ $\alpha=\beta$ or $\alpha=\gamma$ or $\beta=\gamma$: pair 453.5 $\boxed{B}$ $\alpha=0$ or $\beta=0$ or $\gamma=0$: pair 448 $\boxed{B}$ $\alpha=\infty$ or $\beta=\infty$ or $\gamma=\infty$: pair 449	$\dfrac{\alpha(\beta-\gamma)e^{-\alpha g}+\beta(\gamma-\alpha)e^{-\beta g}+\gamma(\alpha-\beta)e^{-\gamma g}}{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}, \quad 0 < g$
453.5	$\dfrac{p}{(p+\alpha)(p+\beta)^2}$ $\boxed{S}$ $\alpha = \beta$: pair 453.8 $\boxed{B}$ $\alpha = 0$: pair 442 $\boxed{B}$ $\alpha = \infty$: pair 449.5	$\dfrac{[\alpha - \beta(\alpha - \beta)g]e^{-\beta g} - \alpha e^{-\alpha g}}{(\alpha - \beta)^2}, \quad 0 < g$
453.8	$\dfrac{p}{(p+\beta)^3}$	$g(1 - \frac{1}{2}\beta g)e^{-\beta g}, \quad 0 < g$
454	$\dfrac{1}{p(p+\alpha)(p+\beta)} - \dfrac{1}{\alpha\beta p}$ $\boxed{S}$ $\alpha = \beta$: pair 454.5 $\boxed{B}$ $\alpha = \infty$ or $\beta = \infty$: pair 438	$\dfrac{\beta e^{-\alpha g} - \alpha e^{-\beta g}}{\alpha\beta(\alpha - \beta)}, \quad 0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
454.5	$\dfrac{1}{p(p+\beta)^2}-\dfrac{1}{\beta^2 p}$	$-\dfrac{1}{\beta^2}(1+\beta g)e^{-\beta g}$	$0<g$
455.4	$\dfrac{1}{p(p^2+\kappa^2)}$ $=\lim\limits_{\beta\to 0}\left\{\dfrac{1}{(p+\beta)[(p+\beta)^2+\kappa^2]}\right\}$	$\dfrac{2}{\kappa^2}\sin^2(\frac{1}{2}\kappa g),$	$0<g$
460.3	$\dfrac{1}{(p+\beta)[(p+\beta)^2+\lambda^2]}$ $\boxed{S}$ $\lambda=0$ pair 450 $\boxed{S}$ $\beta=\infty$ and $\lambda=\infty$ pair 438 $\boxed{D}$ $\beta\to 0$ pair 455.4 $\boxed{C}$ $R(\beta)\geqq\lvert I(\lambda)\rvert$	$\dfrac{2}{\lambda^2}\sin^2(\frac{1}{2}\lambda g)e^{-\beta g},$	$0<g$
	Part 5 Irrational Algebraic Functions of f		
501	$p^{n-\alpha}=\lim\limits_{\beta\to 0}(p^{n-\alpha}e^{-\beta\lvert p\rvert})$ $\boxed{S}$ $n=0$ pair 521 $\boxed{S}$ $\alpha=\frac{1}{2}$ pair 502.1 $\boxed{C}$ $n+1\leqq R(\alpha)$ $\boxed{E}$ This is a regular pair (521) if $n<R(\alpha)$	$\dfrac{1}{\Gamma(\alpha-n)}g^{\alpha-n-1},$	$0<g$
502	$p^{k-\frac{1}{2}}=\lim\limits_{\beta\to 0}(p^{k-\frac{1}{2}}e^{-\beta\lvert p\rvert})$ $\boxed{S}$ $k=1,2$ pairs 503, 504	$\dfrac{(-1)^k 1\cdot 3\cdot 5\cdots(2k-1)}{2^k\pi^{\frac{1}{2}}g^{k+\frac{1}{2}}},$	$0<g$
502.1	$p^{n-\frac{1}{2}}=\lim\limits_{\beta\to 0}(p^{n-\frac{1}{2}}e^{-\beta\lvert p\rvert})$ $\boxed{S}$ $n=k$ pair 502 $\boxed{S}$ $n=1, 2$ pairs 503, 504 $\boxed{E}$ This is a regular pair (522) if $n=0$	$\dfrac{(-1)^n\Gamma(n+\frac{1}{2})}{\pi g^{n+\frac{1}{2}}}=\dfrac{(-1)^n(2n)!}{2^{2n}n!\pi^{\frac{1}{2}}g^{n+\frac{1}{2}}},$	$0<g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
503	$p^{\frac{1}{2}} = \lim\limits_{\beta\to 0} (p^{\frac{1}{2}} e^{-\beta\lvert p\rvert})$	$-\dfrac{1}{2\pi^{\frac{1}{2}} g^{\frac{3}{2}}},$	$0 < g$
503.1	$p^{\frac{1}{2}} = \lim\limits_{\alpha\to 0} [p^{\frac{1}{2}} \exp(-\alpha^{\frac{1}{2}} p^{\frac{1}{2}})]$	$-\dfrac{1}{2\pi^{\frac{1}{2}} g^{\frac{3}{2}}},$	$0 < g$
504	$p^{\frac{3}{2}} = \lim\limits_{\beta\to 0} (p^{\frac{3}{2}} e^{-\beta\lvert p\rvert})$	$\dfrac{3}{4\pi^{\frac{1}{2}} g^{\frac{5}{2}}},$	$0 < g$
505.1	$\lvert p\rvert^{\frac{1}{2}} = \lim\limits_{\beta\to 0} (\lvert p\rvert^{\frac{1}{2}} e^{-\beta\lvert p\rvert})$	$-\dfrac{1}{2^{\frac{3}{2}}\pi^{\frac{1}{2}} \lvert g\rvert^{\frac{3}{2}}}$	
506	$(p+\rho)^{\frac{1}{2}} = \lim\limits_{\alpha\to 0} \{(p+\rho)^{\frac{1}{2}} \exp[-\alpha^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]\}$ $\boxed{S}$ $\rho = 0$: pair 503.1	$-\dfrac{1}{2\pi^{\frac{1}{2}} g^{\frac{3}{2}}} e^{-\rho g},$	$0 < g$
516	$\dfrac{1}{p^{\alpha}} = \lim\limits_{\beta\to 0} \left[\dfrac{1}{(p+\beta)^{\alpha}}\right]$ $\boxed{S}$ $\alpha = k + \frac{1}{2}$: pair 518 $\boxed{S}$ $\alpha = \frac{3}{2}$: pair 520 $\boxed{E}$ This is a regular pair (521) if $R(\alpha) < 1$	$\dfrac{1}{\Gamma(\alpha)} g^{\alpha-1},$	$0 < g$
517	$\dfrac{1}{p^{\alpha}} = \lim\limits_{\beta\to 0} \left[\dfrac{1}{(p-\beta)^{\alpha}}\right]$ $\boxed{M}$ br$(-\frac{1}{2})$ $\boxed{E}$ This is a regular pair (521.1) if $R(\alpha) < 1$	$-\dfrac{1}{\Gamma(\alpha)} g^{\alpha-1},$	$g < 0$
518	$\dfrac{1}{p^{k+\frac{1}{2}}} = \lim\limits_{\beta\to 0} \left[\dfrac{1}{(p+\beta)^{k+\frac{1}{2}}}\right]$ $\boxed{S}$ $k = 1$: pair 520	$\dfrac{2^k}{1\cdot 3\cdot 5 \cdots (2k-1)\pi^{\frac{1}{2}}} g^{k-\frac{1}{2}},$	$0 < g$
519	$\dfrac{1}{p^{k+\frac{1}{2}}} = \lim\limits_{\beta\to 0} \left[\dfrac{1}{(p-\beta)^{k+\frac{1}{2}}}\right]$ $\boxed{M}$ br $\frac{1}{2}$	$\dfrac{2^k}{1\cdot 3\cdot 5 \cdots (2k-1)\pi^{\frac{1}{2}}} g^{k-\frac{1}{2}},$	$g < 0$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
520	$\frac{1}{p^{3/2}} = \lim_{\beta \to 0} \left[\frac{1}{(p+\beta)^{3/2}}\right]$	$\frac{2g^{1/2}}{\pi^{1/2}}$	$0 < g$
521	$\frac{1}{p^{\alpha}}$ ▣ $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ pairs 521.4, 522, 521.7 ▣ $1 \leqq R(\alpha)$	$\frac{1}{\Gamma(\alpha)} g^{\alpha-1}$,	$0 < g$
521.1	$\frac{1}{p^{\alpha}}$ ▣ br$(-\frac{1}{2})$ ▣ $\alpha = \frac{1}{2}$ pair 522.1 ▣ $1 \leqq R(\alpha)$	$-\frac{1}{\Gamma(\alpha)} g^{\alpha-1}$,	$g < 0$
521.4	$\frac{1}{p^{1/4}}$	$\frac{1}{\Gamma(\frac{1}{4}) g^{3/4}}$,	$0 < g$
521.7	$\frac{1}{p^{3/4}}$	$\frac{1}{\Gamma(\frac{3}{4}) g^{1/4}}$,	$0 < g$
522	$\frac{1}{p^{1/2}}$	$\frac{1}{(\pi g)^{1/2}}$,	$0 < g$
522.1	$\frac{1}{p^{1/2}}$ ▣ br$(\pm \frac{1}{2})$	$\pm \frac{1}{(\pi g)^{1/2}}$,	$g < 0$
522.2	$\frac{\lvert p \rvert}{p^{3/2}}$	$-\frac{1}{(\pi \lvert g \rvert)^{1/2}}$,	$g < 0$
522.5	$\frac{1}{\lvert p \rvert^{\alpha}}$ ▣ $\alpha = \frac{1}{2}$ pair 523.1 ▣ $1 \leqq R(\alpha)$	$\frac{\lvert g \rvert^{\alpha-1}}{2\Gamma(\alpha) \cos(\frac{1}{2}\pi\alpha)}$	

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$	
522.8	$\dfrac{\lvert p\rvert}{p^{\alpha+1}}$ $\boxed{S}$ $\alpha = \frac{1}{2}$: pair 522.2 $\boxed{C}$ $1 \leqq R(\alpha)$	$-\dfrac{\lvert g\rvert^{\alpha-1}}{\sin \pi\alpha\, \Gamma(\alpha)}$,	$g < 0$
		$\dfrac{\lvert g\rvert^{\alpha-1}}{\tan \pi\alpha\, \Gamma(\alpha)}$,	$0 < g$
523	$\dfrac{1}{\lvert f\rvert^{\frac{1}{2}}}$ $\boxed{A}$ pair 523.1	$\dfrac{1}{\lvert g\rvert^{\frac{1}{2}}}$	
523.1	$\dfrac{1}{\lvert p\rvert^{\frac{1}{2}}}$ $\boxed{A}$ pair 523	$\dfrac{1}{(2\pi\lvert g\rvert)^{\frac{1}{2}}}$	
523.5	$\dfrac{\lvert p\rvert^{\frac{1}{2}}}{p}$	$\dfrac{\pm 1}{(2\pi\lvert g\rvert)^{\frac{1}{2}}}$,	$0 < \pm g$
524	$\dfrac{1}{(p+\rho)^{\alpha}}$ $\boxed{M}$ br v $\boxed{S}$ $v = w = 0$: pair 524.2 $\boxed{C}$ $1 \leqq R(\alpha)$, $R(\rho) = 0$	$\dfrac{1}{\Gamma(\alpha)} g^{\alpha-1} e^{-(2\pi\alpha(v+w)-\rho g)}$, $\boxed{M}$ br w	$0 < g$
524.2	$\dfrac{1}{(p+\beta)^{\alpha}}$ $\boxed{S}$ $\alpha = k$: pair 431 $\boxed{S}$ $\alpha = n + \frac{1}{2}$: pair 524.5 $\boxed{S}$ $\alpha = k + \frac{1}{2}$: pair 524.6 $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$: pairs 526.4, 526, 526.7, 438, 529, 442, 529.3, 450. $\boxed{B}$ $\beta = 0$: pair 521 $\boxed{L}$ $\beta \to 0$: pair 516 $\boxed{C}$ $R(\alpha) < 1$; $R(\beta) = 0$	$\dfrac{1}{\Gamma(\alpha)} g^{\alpha-1} e^{-\beta g}$,	$0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
524 5	$\dfrac{1}{(p+\beta)^{n+\frac{1}{2}}}$ ■ $n = k$ pa r 524 6 ■ $n = 1 \; 2$ pa rs 529 529.3 ■ $n = 0$ pa r 526	$\dfrac{2^{2n}\, n!}{\pi^{\frac{1}{2}}(2n)!}\, g^{n-\frac{1}{2}} e^{-\beta g}$	$0 < g$
524 6	$\dfrac{1}{(p+\beta)^{k+\frac{1}{2}}}$ ■ $k = 1 \; 2$ pa rs 529 529 3 ■ $\beta \to 0$ pa r 518	$\dfrac{2^{k}}{1 \; 3 \; 5 \quad (2k-1)\pi^{\frac{1}{2}}}\, g^{k-\frac{1}{2}} e^{-\beta g}$	$0 < g$
525	$\dfrac{1}{(p-\rho)^{\alpha}}$ ■ br $(v - \frac{1}{2})$ ■ $\alpha = n + \frac{1}{2}$ pa r 525 5 ■ $\alpha = k + \frac{1}{2}$ pa r 525 6 ■ $\alpha = \frac{1}{2}$ pa r 527 ■ $v = w = 0$ pa r 523 2 ■ $1 \geqq R(\alpha) \;\; R(\rho) = 0$	$-\dfrac{1}{\Gamma(\alpha)}\, g^{\alpha-1} e^{\,[illegible]}$ ■ br w	$g < 0$
523 2	$\dfrac{1}{(p-\beta)^{\alpha}}$ ■ br $(-\frac{1}{2})$ ■ $\alpha = k$ pa r 431 ■ $\alpha = \frac{1}{2} \; 1 \; 2 \; 3$ pa rs 527 439 442 450 ■ $\beta = 0$ pa r 521 1 ■ $\beta \to 0$ pa r 517 ■ $R(\alpha) < 1 \;\; R(\beta) = 0$	$-\dfrac{1}{\Gamma(\alpha)}\, g^{\alpha-1} e^{\beta g}$	$g < 0$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$	
525.5	$\dfrac{1}{(p-\beta)^{n+\frac{1}{2}}}$ $\boxed{\text{M}}$ br $\frac{1}{2}$ $\boxed{\text{S}}$ $n = k$: pair 525.6 $\boxed{\text{B}}$ $n = 0$: pair 527	$\dfrac{2^{2n}n!}{\pi^{\frac{1}{2}}(2n)!}g^{n-\frac{1}{2}}e^{\beta g},$	$g < 0$
525.6	$\dfrac{1}{(p-\beta)^{k+\frac{1}{2}}}$ $\boxed{\text{M}}$ br $\frac{1}{2}$ $\boxed{\text{L}}$ $\beta \to 0$: pair 519	$\dfrac{2^k}{1\cdot 3\cdot 5\cdots(2k-1)\pi^{\frac{1}{2}}}g^{k-\frac{1}{2}}e^{\beta g},$	$g < 0$
526	$\dfrac{1}{(p+\rho)^{\frac{1}{2}}}$ $\boxed{\text{S}}$ $\rho = 0$: pair 522	$\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{-\rho g},$	$0 < g$
526.4	$\dfrac{1}{(p+\rho)^{\frac{1}{4}}}$ $\boxed{\text{S}}$ $\rho = 0$: pair 521.4	$\dfrac{1}{\Gamma(\frac{1}{4})g^{\frac{3}{4}}}e^{-\rho g},$	$0 < g$
526.7	$\dfrac{1}{(p+\rho)^{\frac{3}{4}}}$ $\boxed{\text{S}}$ $\rho = 0$: pair 521.7	$\dfrac{1}{\Gamma(\frac{3}{4})g^{\frac{1}{4}}}e^{-\rho g},$	$0 < g$
527	$\dfrac{1}{(p-\rho)^{\frac{1}{2}}}$ $\boxed{\text{M}}$ br $(\pm\frac{1}{2})$ $\boxed{\text{S}}$ $\rho = 0$: pair 522.1	$\pm\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{\rho g},$	$g < 0$
528	$\dfrac{1}{(p-r)^{\frac{1}{2}}}$ $\boxed{\text{S}}$ $r = 0$: pair 522	$\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{rg}\operatorname{erfc}(r^{\frac{1}{2}}g^{\frac{1}{2}}),$	$0 < g$
		$-\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{rg}\operatorname{erf}(r^{\frac{1}{2}}g^{\frac{1}{2}}),$	$g < 0$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
529	$\dfrac{1}{(p+\beta)^{3/2}}$ $\boxed{L}$ $\beta \to 0$ pair 520	$\dfrac{2g^{1/2}}{\pi^{1/2}} e^{-\beta g}$,	$0 < g$
529.3	$\dfrac{1}{(p+\beta)^{5/2}}$	$\dfrac{4g^{3/2}}{3\pi^{1/2}} e^{-\beta g}$,	$0 < g$
529.5	$\dfrac{p}{(p+\beta)^{3/2}}$ $\boxed{S}$ $\beta = 0$ pair 522	$\dfrac{1}{(\pi g)^{1/2}} e^{-\beta g}(1 - 2\beta g)$,	$0 < g$
530	$\dfrac{1}{(p+\rho)^{\alpha-1}} - \dfrac{1}{(p+\sigma)^{\alpha-1}}$ $\boxed{S}$ $\alpha = \frac{1}{2},\ 1,\ \frac{3}{2},\ 2$ pairs 530.5, 894, 530.3, 448 $\boxed{S}$ $\rho = 0$ or $\sigma = 0$ pair 531.1 $\boxed{L}$ $\rho = \infty$ or $\sigma = \infty$ pair 524.2 $\boxed{L}$ $\rho = \sigma$ pair 524.2 $\boxed{E}$ $2 \leqq R(\alpha)$, $R(\rho) = 0$ or $R(\sigma) = 0$	$\dfrac{1}{\Gamma(\alpha - 1)} g^{\alpha-2}(e^{-\rho g} - e^{-\sigma g})$,	$0 < g$
530.3	$\dfrac{1}{(p+\rho)^{1/2}} - \dfrac{1}{(p+\sigma)^{1/2}}$ $\boxed{S}$ $\rho = 0$ or $\sigma = 0$ pair 531.3 $\boxed{L}$ $\rho = \infty$ or $\sigma = \infty$ pair 526 $\boxed{L}$ $\rho = \sigma$ pair 529	$\dfrac{1}{(\pi g)^{1/2}}(e^{-\rho g} - e^{-\sigma g})$,	$0 < g$
530.5	$(p+\rho)^{1/2} - (p+\sigma)^{1/2}$ $\boxed{S}$ $\rho = \sigma$ pair 526 $\boxed{S}$ $\rho = -\sigma$ pair 556.5 $\boxed{S}$ $\rho = 0$ or $\sigma = 0$ pair 531.5	$\dfrac{1}{2\pi^{1/2}g^{3/2}}(e^{-\sigma g} - e^{-\rho g})$,	$0 < g$
531.1	$\dfrac{1}{(p+\rho)^{\alpha-1}} - \dfrac{1}{p^{\alpha-1}}$ $\boxed{S}$ $\alpha = \frac{1}{2},\ 1,\ \frac{3}{2}$ pairs 531.5, 894.2, 531.3 $\boxed{L}$ $\rho = \infty$ pair 521 $\boxed{L}$ $\rho = 0$ pair 521 $\boxed{E}$ $2 \leqq R(\alpha)$	$\dfrac{1}{\Gamma(\alpha - 1)} g^{\alpha-2}(e^{-\rho g} - 1)$	$0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
531.3	$\dfrac{1}{(p+\rho)^{1/2}} - \dfrac{1}{p^{1/2}}$ $\boxed{\text{B}}$ $\rho = \infty$: pair 522 $\boxed{\text{O}}$ $\rho = 0$	$\dfrac{1}{(\pi g)^{1/2}}(e^{-\rho g} - 1)$,	$0 < g$
531.5	$(p+\rho)^{1/2} - p^{1/2}$ $\boxed{\text{S}}$ $\rho = 0$: pair 522	$\dfrac{1}{2\pi^{1/2}g^{3/2}}(1 - e^{-\rho g})$,	$0 < g$
539.1	$\dfrac{1}{p^{1/4}(p+\rho)^{1/4}}$ $\boxed{\text{S}}$ $\rho = 0$: pair 522 $\boxed{\text{B}}$ $\rho = \infty$: pair 521.4	$\dfrac{\Gamma(\frac{3}{4})}{(2\pi)^{1/2}}\left(\dfrac{\rho}{g}\right)^{1/4} e^{-\frac{1}{2}\rho g} I_{-\frac{1}{4}}(\tfrac{1}{2}\rho g)$,	$0 < g$
539.7	$\dfrac{1}{p^{3/4}(p+\rho)^{3/4}}$ $\boxed{\text{B}}$ $\rho = \infty$: pair 521.7 $\boxed{\text{O}}$ $\rho = 0$	$\dfrac{\Gamma(\frac{1}{4})}{(2\pi)^{1/2}}\left(\dfrac{g}{\rho}\right)^{1/4} e^{-\frac{1}{2}\rho g} I_{\frac{1}{4}}(\tfrac{1}{2}\rho g)$,	$0 < g$
540.1	$\dfrac{p^{\alpha-1}}{p+\beta}$ $\boxed{\text{S}}$ $\alpha = \frac{1}{2}, 1, \frac{3}{2}$: pairs 542, 438, 541 $\boxed{\text{B}}$ $\beta = 0$: pair 521 $\boxed{\text{B}}$ $\beta = \infty$: pair 521 $\boxed{\text{O}}$ $2 \leqq R(\alpha)$	$\dfrac{i^{2\alpha-2}\beta^{\alpha-1}}{\Gamma(1-\alpha)} e^{-\beta g}\gamma(1-\alpha, i^2\beta g)$,	$0 < g$
541	$\dfrac{p^{1/2}}{p+\beta}$ $\boxed{\text{B}}$ $\beta = 0$: pair 522	$\dfrac{1}{(\pi g)^{1/2}} + i\beta^{1/2}e^{-\beta g}\operatorname{erf}(i\beta^{1/2}g^{1/2})$,	$0 < g$
542	$\dfrac{1}{p^{1/2}(p+\beta)}$ $\boxed{\text{B}}$ $\beta = \infty$: pair 522	$\dfrac{1}{i\beta^{1/2}} e^{-\beta g}\operatorname{erf}(i\beta^{1/2}g^{1/2})$,	$0 < g$
543	$\dfrac{1}{1+\beta^{1/2}p^{1/2}}$ $\boxed{\text{B}}$ $\beta = \infty$: pair 522	$\dfrac{1}{\pi^{1/2}\beta^{1/2}g^{1/2}} - \dfrac{1}{\beta}\exp\left(\dfrac{g}{\beta}\right)\operatorname{erfc}\left(\dfrac{g^{1/2}}{\beta^{1/2}}\right)$,	$0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
543 5	$\dfrac{1}{\lambda + (p + \rho)^{1/2}}$ $\rho = 0$ pair 543 $\lambda = 0$ pair 526 $R(\lambda) \geqq -\{R(\rho) + [I(\lambda)]^2\}^{1/2}$	$e^{-\rho g}\left[\dfrac{1}{(\pi g)^{1/2}} - \lambda \exp(\lambda^2 g) \operatorname{erfc}(\lambda g^{1/2})\right]$, $0 < g$
545	$\dfrac{p}{(p + \gamma)(1 + \beta^{1/2} p^{1/2})}$ $\beta = \infty$ pair 541 $\gamma = 0$ pair 543	$\dfrac{1}{1 + \beta^2\gamma}\left[i\beta^{1/2}\gamma^{1/2} e^{-\gamma g} \operatorname{erf}(i\gamma^{1/2} g^{1/2}) - \gamma e^{-\gamma g} - \dfrac{1}{\beta^2}\exp\left(\dfrac{g}{\beta^2}\right)\operatorname{erfc}\left(\dfrac{g^{1/2}}{\beta}\right)\right] + \dfrac{1}{\pi^{1/2}\beta g^{1/2}}$, $0 < g$
545 2	$\dfrac{p + \rho}{(p + \gamma)[\lambda + (p + \rho)^{1/2}]}$ $\rho = 0$ pair 545 $\gamma = \rho$ pair 543.5 $\lambda = \infty$ or $\rho = \infty$ pair 438 $\lambda = 0$ pair 549 $R(\lambda) \geqq -\{R(\rho) + [I(\lambda)]^2\}^{1/2}$	$\dfrac{(\rho - \gamma)e^{-\gamma g}}{\lambda^2 + \gamma - \rho}\{\lambda - (\rho - \gamma)^{1/2} \times \operatorname{erf}[(\rho - \gamma)^{1/2} g^{1/2}]\} + e^{-\rho g}\left[\dfrac{1}{(\pi g)^{1/2}} - \dfrac{\lambda^3 \exp(\lambda^2 g)\operatorname{erfc}(\lambda g^{1/2})}{\lambda^2 + \gamma - \rho}\right]$, $0 < g$
545 5	$\dfrac{1}{(p + \gamma)(1 + \beta^{1/2} p^{1/2})}$ $\beta = 0$ pair 438 $\beta = \infty$ pair 542 $\gamma = \infty$ pair 543	$\dfrac{1}{1 + \beta^2\gamma}\left[e^{-\gamma g} - i\beta^{1/2}\gamma^{1/2} e^{-\gamma g} \operatorname{erf}(i\gamma^{1/2} g^{1/2}) - \exp\left(\dfrac{g}{\beta^2}\right)\operatorname{erfc}\left(\dfrac{g^{1/2}}{\beta}\right)\right]$, $0 < g$
545 7	$\dfrac{1}{(p + \gamma)[\lambda + (p + \rho)^{1/2}]}$ $\rho = 0$ pair 545 5 $\gamma = \infty$ pair 543 5 $\lambda = \infty$ or $\rho = \infty$ pair 438 $\lambda = 0$ pair 546 $R(\lambda) \geqq -\{R(\rho) + [I(\lambda)]^2\}^{1/2}$	$\dfrac{1}{\lambda^2 + \gamma - \rho}\{\lambda e^{-\gamma g} - (\rho - \gamma)^{1/2} e^{-\gamma g} \times \operatorname{erf}[(\rho - \gamma)^{1/2} g^{1/2}] - \lambda \exp(\lambda^2 g - \rho g)\operatorname{erfc}(\lambda g^{1/2})\}$, $0 < g$
545 9	$\dfrac{1}{(p + \beta)[\lambda + (p + \beta)^{1/2}]}$ $\lambda = 0$ pair 529 $\lambda = \infty$ pair 438 $R(\lambda) \geqq -\{R(\beta) + [I(\lambda)]^2\}^{1/2}$	$\dfrac{e^{-\beta g}}{\lambda}[1 - \exp(\lambda^2 g)\operatorname{erfc}(\lambda g^{1/2})]$ $\quad 0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
546	$\dfrac{1}{(p+\beta)(p+\rho)^{\frac{1}{2}}}$ $\boxed{S}$ $\rho=\beta$: pair 529 $\boxed{S}$ $\rho=0$: pair 542 $\boxed{B}$ $\beta=\infty$: pair 526 $\boxed{B}$ $\rho=\infty$: pair 438	$\dfrac{1}{(\rho-\beta)^{\frac{1}{2}}}e^{-\beta g}\operatorname{erf}[(\rho-\beta)^{\frac{1}{2}}g^{\frac{1}{2}}]$, $\quad 0<g$
547.1	$\dfrac{1}{p(p+\rho)^{\frac{1}{2}}}-\dfrac{1}{\rho^{\frac{1}{2}}p}$ $\boxed{O}$ $\rho=0$	$-\dfrac{1}{\rho^{\frac{1}{2}}}\operatorname{erfc}(\rho^{\frac{1}{2}}g^{\frac{1}{2}})$, $\quad 0<g$
548.1	$\dfrac{(p+\rho)^{\frac{1}{2}}}{p}-\dfrac{\rho^{\frac{1}{2}}}{p}$ $\boxed{S}$ $\rho=0$: pair 522	$\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{-\rho g}-\rho^{\frac{1}{2}}\operatorname{erfc}(\rho^{\frac{1}{2}}g^{\frac{1}{2}})$, $\quad 0<g$
549	$\dfrac{(p+\rho)^{\frac{1}{2}}}{p+\beta}$ $\boxed{S}$ $\rho=0$: pair 541 $\boxed{B}$ $\beta=\rho$: pair 526 $\boxed{B}$ $\rho=\infty$: pair 438	$\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{-\rho g}+(\rho-\beta)^{\frac{1}{2}}e^{-\beta g}\operatorname{erf}[(\rho-\beta)^{\frac{1}{2}}g^{\frac{1}{2}}]$, $0<g$
551	$\dfrac{1}{p^{\frac{1}{2}}(1+\beta^{\frac{1}{2}}p^{\frac{1}{2}})}$ $\boxed{B}$ $\beta=0$: pair 522	$\dfrac{1}{\beta^{\frac{1}{2}}}\exp\left(\dfrac{g}{\beta^{3}}\right)\operatorname{erfc}\left(\dfrac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right)$, $\quad 0<g$
551.5	$\dfrac{1}{(p+\rho)^{\frac{1}{2}}[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\boxed{S}$ $\lambda=0$: pair 438 $\boxed{S}$ $\lambda^2=\rho$: pair 547.1 $\boxed{S}$ $\rho=0$: pair 551 $\boxed{B}$ $\lambda=\infty$: pair 526 $\boxed{O}$ $R(\lambda)\leqq-\{R(\rho)+[I(\lambda)]^2\}^{\frac{1}{2}}$	$\exp(\lambda^2 g-\rho g)\operatorname{erfc}(\lambda g^{\frac{1}{2}})$, $\quad 0<g$
552	$\dfrac{p^{\frac{1}{2}}}{(p+\gamma)(1+\beta^{\frac{1}{2}}p^{\frac{1}{2}})}$ $\boxed{B}$ $\beta=0$: pair 541 $\boxed{B}$ $\beta=\infty$: pair 438 $\boxed{B}$ $\gamma=0$: pair 551	$\dfrac{1}{1+\beta^{\frac{1}{2}}\gamma}\Big[i\gamma^{\frac{1}{2}}e^{-\gamma g}\operatorname{erf}(i\gamma^{\frac{1}{2}}g^{\frac{1}{2}})+\beta^{\frac{1}{2}}\gamma e^{-\gamma g}$ $+\dfrac{1}{\beta^{\frac{1}{2}}}\exp\left(\dfrac{g}{\beta^{3}}\right)\operatorname{erfc}\left(\dfrac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right)\Big]$, $\quad 0<g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
552.1	$\dfrac{(p+\rho)^{\frac{1}{2}}}{(p+\gamma)[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\boxtimes$ $\rho = 0$ pair 552 $\boxtimes$ $\gamma = \rho$ pair 551.5 $\boxtimes$ $\lambda = \infty$ pair 549 $\boxtimes$ $\rho = \infty$ pair 438 $\boxtimes$ $\lambda = 0$ pair 438 $\boxtimes$ $R(\lambda) \gtrless -\{R(\rho) + [I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{1}{\lambda^2+\gamma-\rho}\{\lambda(\rho-\gamma)^{\frac{1}{2}}e^{-\gamma g}$ $\times \operatorname{erf}[(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}]-(\rho-\gamma)e^{-\gamma g}$ $+\lambda^2\exp(\lambda^2 g-\rho g)\operatorname{erfc}(\lambda g^{\frac{1}{2}})\}, \quad 0 < g$
552.3	$\dfrac{1}{p^{\frac{1}{2}}(p+\gamma)(1+\beta^{\frac{1}{2}}p^{\frac{1}{2}})}$ $\boxtimes$ $\beta = 0$ pair 542 $\boxtimes$ $\gamma = \infty$ pair 551	$\dfrac{1}{1+\beta^2\gamma}\left[\dfrac{e^{-\gamma g}}{i\gamma^{\frac{1}{2}}}\operatorname{erf}(i\gamma^{\frac{1}{2}}g^{\frac{1}{2}}) - \beta^{\frac{1}{2}}e^{-\gamma g}\right.$ $\left. + \beta^{\frac{1}{2}}\exp\left(\dfrac{g}{\beta^2}\right)\operatorname{erfc}\left(\dfrac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right)\right], \quad 0 < g$
552.5	$\dfrac{1}{(p+\rho)^{\frac{1}{2}}(p+\gamma)[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\boxtimes$ $\lambda = 0$ pair 448 $\boxtimes$ $\rho = 0$ pair 552.3 $\boxtimes$ $\gamma = \infty$ pair 551.5 $\boxtimes$ $\lambda = \infty$ pair 546 $\boxtimes$ $\rho = \infty$ pair 438 $\boxtimes$ $R(\lambda) \gtrless -\{R(\rho) + [I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{1}{\lambda^2+\gamma-\rho}\left\{\dfrac{\lambda e^{-\gamma g}}{(\rho-\gamma)^{\frac{1}{2}}}\operatorname{erf}[(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}]\right.$ $\left. - e^{-\gamma g} + \exp(\lambda^2 g-\rho g)\operatorname{erfc}(\lambda g^{\frac{1}{2}})\right\},$ $0 < g$
552.9	$\dfrac{1}{(p+\beta)^{\frac{1}{2}}[\lambda+(p+\beta)^{\frac{1}{2}}]}$ $\boxtimes$ $\lambda = 0$ pair 442 $\boxtimes$ $\lambda = \infty$ pair 529 $\boxtimes$ $R(\lambda) \gtrless -\{R(\beta) + [I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{e^{-\beta g}}{\lambda^2}\left[\exp(\lambda^2 g)\operatorname{erfc}(\lambda g^{\frac{1}{2}}) - 1 + \dfrac{2\lambda g^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\right],$ $0 < g$
553.1	$\left(\dfrac{p+\rho}{p}\right)^{\frac{1}{2}} - 1$ $\boxtimes$ $\rho = \infty$ pair 522 $\boxtimes$ $\rho = 0$	$\frac{1}{2}\rho e^{-\frac{1}{2}\rho g}[I_1(\frac{1}{2}\rho g) + I_0(\frac{1}{2}\rho g)], \quad 0 < g$
553.5	$\left(\dfrac{p}{p+\rho}\right)^{\frac{1}{2}} - 1$ $\boxtimes$ $\rho = 0$	$\frac{1}{2}\rho e^{-\frac{1}{2}\rho g}[I_0(\frac{1}{2}\rho g) - I_1(\frac{1}{2}\rho g)] \quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
554	$\dfrac{(p+\rho)^{1/2}}{p^{3/2}} = \lim\limits_{\beta\to 0}\left[\dfrac{(p+\rho)^{1/2}}{(p+\beta)^{3/2}}\right]$ $\boxed{B}$ $\rho = \infty$: pair 520	$e^{-\frac{1}{2}\rho g}[\rho g I_1(\frac{1}{2}\rho g) + (1+\rho g) I_0(\frac{1}{2}\rho g)]$, $0 < g$
555	$\dfrac{1}{(p+\rho)^{1/2}(p+\sigma)^{1/2}}$ $\boxed{S}$ $\rho = 0$ or $\sigma = 0$: pair 563.4 $\boxed{S}$ $\sigma = -\rho$: pair 557 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 526 $\boxed{R}$ $\sigma = \rho$: pair 438	$e^{-\frac{1}{2}(\rho+\sigma)g} I_0[\frac{1}{2}(\rho-\sigma)g]$, $0 < g$
555.4	$\dfrac{1}{(p+\rho)^{1/2}(p+\beta)^{3/2}}$ $\boxed{S}$ $\rho = \beta$: pair 442 $\boxed{S}$ $\rho = 0$: pair 563.9 $\boxed{R}$ $\beta = \infty$: pair 526 $\boxed{B}$ $\rho = \infty$: pair 529	$g e^{-\frac{1}{2}(\rho+\beta)g}\{I_0[\frac{1}{2}(\rho-\beta)g]$ $+ I_1[\frac{1}{2}(\rho-\beta)g]\}$, $0 < g$
555.7	$\dfrac{(p+\rho)^{1/2}}{(p+\beta)^{3/2}}$ $\boxed{S}$ $\rho = \beta$: pair 438 $\boxed{S}$ $\rho = 0$: pair 563.7 $\boxed{B}$ $\rho = \infty$: pair 529 $\boxed{L}$ $\beta \to 0$: pair 554	$e^{-\frac{1}{2}(\rho+\beta)g}\{(\rho-\beta)g I_1[\frac{1}{2}(\rho-\beta)g]$ $+ [1 + (\rho-\beta)g] I_0[\frac{1}{2}(\rho-\beta)g]\}$, $0 < g$
556.1	$(p^2+x^2)^{1/2} - p$ $\boxed{D}$ $x = 0$	$\dfrac{x}{g} J_1(xg)$, $0 < g$
556.5	$[(p^2+x^2)^{1/2} - p]^{1/2}$ $\boxed{S}$ $x = 0$: pair 522	$\dfrac{\sin(\|x\|g)}{(2\pi)^{1/2} g^{3/2}}$, $0 < g$
557	$\dfrac{1}{(p^2+x^2)^{1/2}}$ $\boxed{D}$ $x = 0$	$J_0(xg)$, $0 < g$

No	Coefficient $F(f)$	Coefficient $G(g)$	
557 4	$\dfrac{1}{(p^2+x^2)^{\frac14}}$	$\dfrac{\Gamma(\frac34)}{2^{\frac14}\pi^{\frac14}}\left(\dfrac{x}{g}\right)^{\frac14} J_{-\frac14}(xg),$	$0 < g$
	$\boxed{5}\ x = 0$ pair 522		
557 7	$\dfrac{1}{(p^2+x^2)^{\frac34}}$	$\dfrac{\Gamma(\frac14)}{2^{\frac34}\pi^{\frac34}}\left(\dfrac{g}{x}\right)^{\frac14} J_{\frac14}(xg),$	$0 < g$
	$\boxed{8}\ x = 0$		
558	$\dfrac{1}{(\rho^2-p^2)^{\frac12}}$	$\dfrac{1}{\pi}K_0(\rho\|g\|)$	
	$\boxed{8}\ \rho = 0$		
558 5	$\dfrac{1}{(\beta^2-p^2)^{\frac32}}$	$\dfrac{1}{\pi\beta}\|g\|K_1(\beta\|g\|)$	
558 8	$\dfrac{1}{(\rho^2-p^2)^{\frac34}}$	$\dfrac{2^{\frac14}\rho^{\frac14}K_{\frac14}(\rho\|g\|)}{\Gamma(\frac34)\pi^{\frac12}\|g\|^{\frac14}}$	
	$\boxed{8}\ \rho = 0$ pair 523 1		
559 1	$\dfrac{(p+\rho)^{\frac12}-p^{\frac12}}{(p+\rho)^{\frac12}+p^{\frac12}}$	$\dfrac{1}{g}e^{-\frac12\rho g}I_1(\frac12\rho g),$	$0 < g$
	$\boxed{8}\ \rho = 0$		
559 2	$\dfrac{(p+\rho)^{\frac12}-(p+\sigma)^{\frac12}}{(p+\rho)^{\frac12}+(p+\sigma)^{\frac12}}$	$\dfrac{1}{g}e^{-\frac12(\rho+\sigma)g}I_1[\frac12(\rho-\sigma)g],$	$0 < g$
	$\boxed{5}\ \rho = -\sigma$ pair 556 1 $\boxed{5}\ \rho = 0$ or $\sigma = 0$ pair 559 1 $\boxed{8}\ \rho = \sigma$ pair 438		
561 0	$\left(\dfrac{p+\rho}{p+\sigma}\right)^{\frac12} - 1$	$\frac12(\rho-\sigma)e^{-\frac12(\rho+\sigma)g}\{I_1[\frac12(\rho-\sigma)g]$ $+ I_0[\frac12(\rho-\sigma)g]\},$	$0 < g$
	$\boxed{5}\ \rho = 0$ pair 553 5 $\boxed{5}\ \sigma = 0$ pair 553 1 $\boxed{8}\ \rho = \infty$ pair 526 $\boxed{8}\ \rho = \sigma$ pair 438		

No.	Coefficient $F(f)$	Coefficient $G(g)$
563.1	$\dfrac{1}{p^{\alpha}(p+\rho)^{\alpha}}$ $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$: pairs 539.1, 563.4, 539.7 $\boxed{B}$ $\rho = \infty$: pair 521 $\boxed{R}$ $\rho = 0$: pair 521 $\boxed{O}$ $1 \leqq R(\alpha)$	$\dfrac{\pi^{\frac{1}{2}}}{\Gamma(\alpha)}\left(\dfrac{g}{\rho}\right)^{\alpha-\frac{1}{2}} e^{-\frac{1}{2}\rho g} I_{\alpha-\frac{1}{2}}(\tfrac{1}{2}\rho g), \qquad 0 < g$
563.4	$\dfrac{1}{p^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}$ $\boxed{B}$ $\rho = \infty$: pair 522 $\boxed{O}$ $\rho = 0$	$e^{-\frac{1}{2}\rho g} I_0(\tfrac{1}{2}\rho g), \qquad 0 < g$
563.7	$\dfrac{p^{\frac{1}{2}}}{(p+\beta)^{\frac{3}{2}}}$	$e^{-\frac{1}{2}\beta g}[\beta g I_1(\tfrac{1}{2}\beta g) + (1-\beta g) I_0(\tfrac{1}{2}\beta g)], \qquad 0 < g$
563.9	$\dfrac{1}{p^{\frac{1}{2}}(p+\beta)^{\frac{3}{2}}}$ $\boxed{B}$ $\beta = \infty$: pair 522	$g e^{-\frac{1}{2}\beta g}[I_0(\tfrac{1}{2}\beta g) - I_1(\tfrac{1}{2}\beta g)], \qquad 0 < g$
565.1	$\dfrac{p+\rho}{(p+\beta)^{\frac{3}{2}}}$ $\boxed{S}$ $\rho = 0$: pair 529.5 $\boxed{B}$ $\beta = \rho$: pair 526 $\boxed{B}$ $\rho = \infty$: pair 529	$\dfrac{1}{(\pi g)^{\frac{1}{2}}} e^{-\beta g}[1 + 2(\rho-\beta)g], \qquad 0 < g$
565.4	$\dfrac{1}{(p+\beta)(p+\gamma)^{\frac{3}{2}}}$ $\boxed{S}$ $\gamma = \beta$: pair 529.3 $\boxed{B}$ $\beta = \infty$: pair 529 $\boxed{B}$ $\gamma = \infty$: pair 438	$\dfrac{1}{(\gamma-\beta)^{\frac{3}{2}}} e^{-\beta g} \operatorname{erf}[(\gamma-\beta)^{\frac{1}{2}} g^{\frac{1}{2}}] - \dfrac{2g^{\frac{1}{2}}}{\pi^{\frac{1}{2}}(\gamma-\beta)} e^{-\gamma g}, \qquad 0 < g$
569.0	$\dfrac{1}{(\rho^2 - p^2)^{\alpha}}$ $\boxed{S}$ $\alpha = k$: pair 433 $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{1}{2}, 1, \frac{3}{2}$: pairs 558.8, 558, 444, 558.5 $\boxed{R}$ $\rho = 0$: pair 522.5 $\boxed{O}$ $1 \leqq R(\alpha)$, $R(\rho) = 0$	$\dfrac{\lvert g\rvert^{\alpha-\frac{1}{2}} K_{\alpha-\frac{1}{2}}(\rho\lvert g\rvert)}{\pi^{\frac{1}{2}}\Gamma(\alpha)(2\rho)^{\alpha-\frac{1}{2}}}$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
570 1	$\dfrac{1}{(p+\rho)^{\alpha}(p+\sigma)^{\alpha}}$ $\alpha = \frac{1}{4}\ \ \frac{1}{2}\ \ \frac{3}{4}\ \ 1\ \ \frac{3}{2}$ pairs 570 3 555 570 5 448 570 7 $\rho = -\sigma$ pair 571 $\rho = 0$ or $\sigma = 0$ pair 563 1 $\rho = \infty$ or $\sigma = \infty$ pair 524.2 $\rho = \sigma$ pair 524 2 $1 \leqq R(\alpha)$ $R(\rho) = 0$ or $R(\sigma) = 0$	$\dfrac{\pi^{\frac{1}{2}}}{\Gamma(\alpha)}\left(\dfrac{g}{\rho-\sigma}\right)^{\alpha-\frac{1}{2}} e^{-\frac{1}{2}(\rho+\sigma)g}$ $\times I_{\alpha-\frac{1}{2}}[\frac{1}{2}(\rho-\sigma)g]$ $0 < g$
570 3	$\dfrac{1}{(p+\rho)^{\frac{1}{4}}(p+\sigma)^{\frac{1}{4}}}$ $\rho = \sigma$ pair 526 $\rho = -\sigma$ pair 557 4 $\rho = 0$ or $\sigma = 0$ pair 539 1 $\rho = \infty$ or $\sigma = \infty$ pair 526 4	$\dfrac{\Gamma(\frac{3}{4})}{(2\pi)^{\frac{1}{2}}}\left(\dfrac{\rho-\sigma}{g}\right)^{\frac{1}{4}} e^{-\frac{1}{2}(\rho+\sigma)g} I_{-\frac{1}{4}}[\frac{1}{2}(\rho-\sigma)g]$ $0 < g$
570 5	$\dfrac{1}{(p+\rho)^{\frac{3}{4}}(p+\sigma)^{\frac{3}{4}}}$ $\rho = -\sigma$ pair 557 7 $\rho = 0$ or $\sigma = 0$ pair 539 7 $\rho = \infty$ or $\sigma = \infty$ pair 526 7 $\sigma = \rho$ pair 529	$\dfrac{\Gamma(\frac{1}{4})}{(2\pi)^{\frac{1}{2}}}\left(\dfrac{g}{\rho-\sigma}\right)^{\frac{1}{4}} e^{-\frac{1}{2}(\rho+\sigma)g} I_{\frac{1}{4}}[\frac{1}{2}(\rho-\sigma)g],$ $0 < g$
570 7	$\dfrac{1}{(p+\beta)^{\frac{3}{2}}(p+\gamma)^{\frac{3}{2}}}$ $\beta = \gamma$ pair 450 $\beta = \infty$ or $\gamma = \infty$ pair 529	$\dfrac{2g}{\beta-\gamma} e^{-\frac{1}{2}(\beta+\gamma)g} I_1[\frac{1}{2}(\beta-\gamma)g],$ $0 < g$
571	$\dfrac{1}{(p^2+x^2)^{\alpha}}$ $\alpha = \frac{1}{4}\ \ \frac{1}{2}\ \ \frac{3}{4}$ pairs 557 4 557 557 7 $x = 0$ pair 521 $1 \leqq R(\alpha)$	$\dfrac{\pi^{\frac{1}{2}}}{\Gamma(\alpha)}\left(\dfrac{g}{2x}\right)^{\alpha-\frac{1}{2}} J_{\alpha-\frac{1}{2}}(xg)$ $0 < g$
573	$\left(\dfrac{p}{p+\rho}\right)^{\frac{1}{2}} \dfrac{1}{[(p+\rho)^{\frac{1}{2}}+\rho^{\frac{1}{2}}]^{2\alpha}}$ $\rho = 0$ pair 521	$\dfrac{1}{4\rho^{\alpha-1}} e^{-\frac{1}{2}\rho g}[I_{\alpha-1}(\frac{1}{2}\rho g) - 2I_{\alpha}(\frac{1}{2}\rho g)$ $+ I_{\alpha+1}(\frac{1}{2}\rho g)]$ $0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
573.5	$\left(\frac{p+\rho}{p+\sigma}\right)^{\frac12}\frac{1}{[(p+\rho)^{\frac12}+(p+\sigma)^{\frac12}]^{2\alpha}}$ $\boxed{S}$ $\rho = 0$: pair 573 $\boxed{B}$ $\rho = \infty$: pair 526 $\boxed{R}$ $\rho = \sigma$: pair 524.2	$\frac{1}{4(\rho-\sigma)^{\alpha-1}}e^{-\frac12(\rho+\sigma)g}$ $\times\{I_{\alpha-1}[\frac12(\rho-\sigma)g]+2I_\alpha[\frac12(\rho-\sigma)g]$ $+I_{\alpha+1}[\frac12(\rho-\sigma)g]\}$, $0 < g$
574	$\frac{[(p+\rho)^{\frac12}+p^{\frac12}]^{2-2\alpha}}{p^{\frac12}(p+\rho)^{\frac12}}$ $\boxed{S}$ $\alpha = 1$: pair 563.4 $\boxed{B}$ $\rho = \infty$: pair 522 $\boxed{R}$ $\rho = 0$: pair 521	$\rho^{1-\alpha}e^{-\frac12\rho g}I_{\alpha-1}(\frac12\rho g)$, $0 < g$
575.1	$\frac{[(p+\rho)^{\frac12}+(p+\sigma)^{\frac12}]^{2-2\alpha}}{(p+\rho)^{\frac12}(p+\sigma)^{\frac12}}$ $\boxed{S}$ $\alpha = 1, \frac32$: pairs 555, 530.3 $\boxed{S}$ $\rho = -\sigma$: pair 575.2 $\boxed{S}$ $\rho = 0$ or $\sigma = 0$: pair 574 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 526 $\boxed{R}$ $\rho = \sigma$: pair 524.2	$(\rho-\sigma)^{1-\alpha}e^{-\frac12(\rho+\sigma)g}I_{\alpha-1}[\frac12(\rho-\sigma)g]$, $0 < g$
575.2	$\frac{[(p^2+x^2)^{\frac12}+p]^{1-\alpha}}{(p^2+x^2)^{\frac12}}$ $\boxed{S}$ $\alpha = 1$: pair 557 $\boxed{R}$ $x = 0$: pair 521	$\frac{J_{\alpha-1}(xg)}{x^{\alpha-1}}$, $0 < g$
576.1	$\frac{1}{[(p+\rho)^{\frac12}+(p+\sigma)^{\frac12}]^{\alpha}}$ $\boxed{S}$ $\alpha = 1, 2$: pairs 530.5, 559.2 $\boxed{S}$ $\rho = -\sigma$: pair 576.3 $\boxed{S}$ $\rho = 0$ or $\sigma = 0$: pair 576.2 $\boxed{R}$ $\rho = \sigma$: pair 524.2	$\frac{\alpha e^{-\frac12(\rho+\sigma)g}I_{\frac12\alpha}[\frac12(\rho-\sigma)g]}{2(\rho-\sigma)^{\frac12\alpha}g}$, $0 < g$
576.2	$\left[\frac{(p+\rho)^{\frac12}-p^{\frac12}}{(p+\rho)^{\frac12}+p^{\frac12}}\right]^{\alpha}$ $\boxed{S}$ $\alpha = \frac12, 1$: pairs 531.5, 559.1 $\boxed{R}$ $\rho = 0$: pair 521	$\frac{\alpha}{g}e^{-\frac12\rho g}I_\alpha(\frac12\rho g)$, $0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
576 3	$\dfrac{1}{[(p^2+x^2)^{\frac12}+p]^{\alpha}}$ $\alpha = \frac12, 1$ pairs 556 5, 556 1 $x = 0$ pair 521	$\dfrac{\alpha}{x^{\alpha}g}J_{\alpha}(xg), \qquad 0 < g$
581 1	$\dfrac{1}{(p+\rho)^{\alpha+\nu}(p+\sigma)^{\alpha-\nu}}$ $\alpha = \frac14, \frac34$ pairs 582 1 582 4 $\alpha = \frac12, \nu = \pm 1$ pair 555 7 $\alpha = 1, \nu = \pm\frac12$ pair 555 4 $\alpha \pm \nu = 1$ pair 581 7 $\rho = 0$ or $\sigma = 0$ pair 581 4 $\nu = 0$ pair 570 1 $\rho = \infty$ or $\sigma = \infty$ pair 524 2 $\alpha \pm \nu = 0$ pair 524 2 $\rho = \sigma$ pair 524 2 $1 \leqq R(\alpha+\nu)$ $R(\rho) = 0$ $1 \leqq R(\alpha-\nu), R(\sigma) = 0$	$\dfrac{1}{\Gamma(2\alpha)(\rho-\sigma)^{\alpha}}g^{\alpha-1}e^{-\frac12(\rho+\sigma)g}$ $\times M_{\nu,\alpha-\frac12}[(\rho-\sigma)g], \quad 0 < g$
581 4	$\dfrac{p^{\beta-1}}{(p+\rho)^{\alpha+\beta-1}}$ $\alpha = \frac12\ \frac32$ pairs 585 1, 585 4 $\beta = 1$ pair 524 2 $\alpha+\beta = 2$ pair 540 1 $\alpha+2\beta = 2$ pair 563 1 $\alpha+\beta = 1$ pair 521 $\rho = \infty$ pair 521 $\rho = 0$ pair 521 $2 \leqq R(\alpha+\beta), R(\rho) = 0$	$\dfrac{1}{\Gamma(\alpha)\rho^{\frac12\alpha}}g^{\frac12\alpha-1}e^{-\frac12\rho g}M_{\frac12\alpha+\beta-1,\ \frac12\alpha-\frac12}(\rho g),$ $0 < g$
581 7	$\dfrac{1}{(p+\beta)(p+\rho)^{\alpha-1}}$ $\alpha = \frac12, \frac32, 2, \frac52$ pairs 549, 546, 448, 565 4 $\rho = 0$ pair 540 1 $\alpha = 1$ pair 438 $\beta = \infty$ pair 524 2 $\rho = \infty$ pair 438 $\beta = \rho$ pair 524 2 $2 \leqq R(\alpha)$ $R(\rho) = 0$	$\dfrac{1}{\Gamma(\alpha-1)(\rho-\beta)^{\alpha-1}}e^{-\beta g}$ $\times \gamma[\alpha-1, (\rho-\beta)g], \quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
582.1	$\dfrac{1}{(p+\rho)^{\frac14+\nu}(p+\sigma)^{\frac14-\nu}}$ S $\nu = 0, \pm\frac34, \pm\frac54$: pairs 570.3, 549, 565.1 S $\rho = 0$ or $\sigma = 0$: pair 585.1 B $\rho = \infty$ or $\sigma = \infty$: pair 524.2 B $\nu = \pm\frac14$: pair 526 R $\rho = \sigma$: pair 526 O $\frac34 \leqq R(\nu)$, $R(\rho) = 0$ O $R(\nu) \leqq -\frac34$, $R(\sigma) = 0$	$\dfrac{1}{\pi^{\frac12}(\rho-\sigma)^{\frac14}g^{\frac34}} e^{-\frac12(\rho+\sigma)g} M_{\nu,-\frac14}[(\rho-\sigma)g]$ $= \dfrac{\Gamma(\frac34-\nu)}{2^{\nu+\frac34}\pi g^{\frac12}} e^{-\frac12(\rho+\sigma)g}$ $\times \{D_{2\nu-\frac12}[-2^{\frac12}(\rho-\sigma)^{\frac12}g^{\frac12}]$ $+ D_{2\nu-\frac12}[2^{\frac12}(\rho-\sigma)^{\frac12}g^{\frac12}]\}, \quad 0 < g$
582.4	$\dfrac{1}{(p+\rho)^{\frac34+\nu}(p+\sigma)^{\frac34-\nu}}$ S $\nu = 0, \pm\frac14$: pairs 570.5, 546 S $\rho = 0$ or $\sigma = 0$: pair 585.4 B $\rho = \infty$ or $\sigma = \infty$: pair 524.2 B $\nu = \pm\frac34$: pair 529 R $\rho = \sigma$: pair 529 O $\frac14 \leqq R(\nu)$, $R(\rho) = 0$ O $R(\nu) \leqq -\frac14$, $R(\sigma) = 0$	$\dfrac{2}{\pi^{\frac12}(\rho-\sigma)^{\frac34}g^{\frac14}} e^{-\frac12(\rho+\sigma)g} M_{\nu,\frac14}[(\rho-\sigma)g]$ $= \dfrac{\Gamma(\frac14-\nu)}{2^{\nu+\frac34}\pi(\rho-\sigma)^{\frac12}} e^{-\frac12(\rho+\sigma)g}$ $\times \{D_{2\nu-\frac12}[-2^{\frac12}(\rho-\sigma)^{\frac12}g^{\frac12}]$ $- D_{2\nu-\frac12}[2^{\frac12}(\rho-\sigma)^{\frac12}g^{\frac12}]\}, \quad 0 < g$
585.1	$\dfrac{p^{\alpha-1}}{(p+\rho)^{\alpha-\frac12}}$ S $\alpha = \frac34, 1, \frac32, 2$: pairs 539.1, 526, 541, 529.5 B $\alpha = \frac12$: pair 522 B $\rho = \infty$: pair 521 B $\rho = 0$: pair 522 O $\frac32 \leqq R(\alpha)$, $R(\rho) = 0$	$\dfrac{1}{\pi^{\frac12}\rho^{\frac14}g^{\frac34}} e^{-\frac12\rho g} M_{\alpha-\frac34,-\frac14}(\rho g)$ $= \dfrac{\Gamma(\frac32-\alpha)}{2^{\alpha}\pi g^{\frac12}} e^{-\frac12\rho g}[D_{2\alpha-2}(-2^{\frac12}\rho^{\frac12}g^{\frac12})$ $+ D_{2\alpha-2}(2^{\frac12}\rho^{\frac12}g^{\frac12})], \quad 0 < g$
585.4	$\dfrac{p^{\alpha-1}}{(p+\rho)^{\alpha+\frac12}}$ S $\alpha = \frac14, \frac12, 1$: pairs 539.7, 542, 529 B $\rho = \infty$: pair 521 O $\frac12 \leqq R(\alpha)$, $R(\rho) = 0$ O $\rho = 0$	$\dfrac{2}{\pi^{\frac12}\rho^{\frac34}g^{\frac14}} e^{-\frac12\rho g} M_{\alpha-\frac14,\frac14}(\rho g)$ $= \dfrac{\Gamma(\frac12-\alpha)}{2^{\alpha+\frac12}\pi\rho^{\frac12}} e^{-\frac12\rho g}[D_{2\alpha-1}(-2^{\frac12}\rho^{\frac12}g^{\frac12})$ $- D_{2\alpha-1}(2^{\frac12}\rho^{\frac12}g^{\frac12})], \quad 0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
	Part 6 Exponential and Trigonometric Functions of f or f [1]		
601	$e^{-xp} = \lim_{\beta\to 0}(e^{-xp-\beta^2 p^2})$	$\mathfrak{S}_0(g-x)$	
	S $x = 0$ pair 403 1		
602	$\frac{e^{-xp}}{p} = \lim_{\beta\to 0}\left[e^{-xp}\left(\frac{\frac{1}{2}-\lambda}{p-\beta}+\frac{\frac{1}{2}+\lambda}{p+\beta}\right)\right]$	$\mathfrak{S}_{-1}(g-x)$	
	S $x = 0$ pair 415		
603	$\frac{1}{p}(e^{\pm ap}-1)$	$\pm 1,$	$0 < \mp g < a$
603 1	$\frac{e^{\pm a(p+\lambda)}-1}{p+\lambda}$	$\pm e^{-\lambda g},$	$0 < \mp g < a$
	S $\lambda = 0$ pair 603 L $a = \infty$ pair 438		
604	$\frac{e^{-xp}}{p\pm\beta}$	$\pm e^{\mp\beta(g-x)},$	$\pm x < \pm g$
	S $x = 0$ pairs 438 and 439		
604 7	$\frac{1}{(p+\beta)[e^{a(p+\beta)}-\mu]}$	$\frac{1-\mu^k}{1-\mu}e^{-\beta g},$	$ka < g < (k+1)a$
	S $\mu = -1$ 0 1 pairs 604 72 604 604 71 L $a = 0$ pair 438 C $aR(\beta) \geqq R(\log\mu)$	R Choice of g fixes k	
604 71	$\frac{1}{(p+\beta)[e^{a(p+\beta)}-1]}$	$ke^{-\beta g},$	$ka < g < (k+1)a$
	L $a = 0$ pair 442	R Choice of g fixes k	
604 72	$\frac{1}{(p+\beta)[e^{a(p+\beta)}+1]}$	$e^{-\beta g},$	$(2k-1)a < g < 2ka$
		R Choice of g fixes k	

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
604.73	$\dfrac{1}{(p+\beta)\sinh[a(p+\beta)]}$ $\boxed{B}$ $a = 0$: pair 442	$2ke^{-\beta g}, \quad (2k-1)a < g < (2k+1)a$ $\boxed{N}$ Choice of g fixes k
604.74	$\dfrac{1}{(p+\beta)\cosh[a(p+\beta)]}$	$2e^{-\beta g}, \quad (4k-3)a < g < (4k-1)a$ $\boxed{N}$ Choice of g fixes k
604.75	$\dfrac{\operatorname{ctnh}[a(p+\beta)]}{p+\beta}$ $\boxed{B}$ $a = 0$: pair 442 $\boxed{B}$ $a = \infty$: pair 438	$(2k-1)e^{-\beta g}, \quad 2a(k-1) < g < 2ak$ $\boxed{N}$ Choice of g fixes k
604.76	$\dfrac{\tanh[a(p+\beta)]}{p+\beta}$ $\boxed{B}$ $a = \infty$: pair 438	$(-1)^{k-1}e^{-\beta g}, \quad 2a(k-1) < g < 2ak$ $\boxed{N}$ Choice of g fixes k
604.8	$\dfrac{1}{(p+\beta)^2[e^{a(p+\beta)}-\mu]}$ $\boxed{S}$ $\mu = -1, 0, 1$: pairs 604.82, 605.1 with $\alpha = 2$, 604.81 $\boxed{B}$ $a = 0$: pair 442 $\boxed{O}$ $aR(\beta) \leqq R(\log \mu)$	$\left[\dfrac{1-\mu^k}{1-\mu}g - \dfrac{1-(k+1)\mu^k+k\mu^{k+1}}{(1-\mu)^2}a\right]e^{-\beta g},$ $ka < g < (k+1)a$ $\boxed{N}$ Choice of g fixes k
604.81	$\dfrac{1}{(p+\beta)^2[e^{a(p+\beta)}-1]}$ $\boxed{B}$ $a = 0$: pair 450	$[kg - \tfrac{1}{2}ak(1+k)]e^{-\beta g},$ $ka < g < (k+1)a$ $\boxed{N}$ Choice of g fixes k
604.82	$\dfrac{1}{(p+\beta)^2[e^{a(p+\beta)}+1]}$ $\boxed{B}$ $a = 0$: pair 442	$\left[\dfrac{1-(-1)^k}{4}(2g-a)+\tfrac{1}{2}ak(-1)^k\right]e^{-\beta g},$ $ka < g < (k+1)a$ $\boxed{N}$ Choice of g fixes k
604.83	$\dfrac{1}{(p+\beta)^2\sinh[a(p+\beta)]}$ $\boxed{B}$ $a = 0$: pair 450	$2k(g-ak)e^{-\beta g}, \; (2k-1)a < g < (2k+1)a$ $\boxed{N}$ Choice of g fixes k

No	Coefficient $F(f)$	Coefficient $G(g)$
604.84	$\dfrac{1}{(p+\beta)^2\cosh[a(p+\beta)]}$	$[g-(-1)^k(g-2ak)]e^{-\beta g}$ $\quad (2k-1)a<g<(2k+1)a$
	$a=0$ pair 442	Choice of g fixes k
604.85	$\dfrac{\operatorname{ctnh}[a(p+\beta)]}{(p+\beta)^2}$	$[(2k-1)g-2ak(k-1)]e^{-\beta g}$ $\quad 2a(k-1)<g<2ak$
	$a=0$ pair 450 $a=\infty$ pair 442	Choice of g fixes k
604.86	$\dfrac{\tanh[a(p+\beta)]}{(p+\beta)^2}$	$[a+(-1)^k(2ak-a-g)]e^{-\beta g}$ $\quad 2a(k-1)<g<2ak$
	$a=0$ pair 438 $a=\infty$ pair 442	Choice of g fixes k
605.1	$\dfrac{e^{-xp}}{(p+\rho)^\alpha}$	$\dfrac{1}{\Gamma(\alpha)}e^{-\rho(g-x)}(g-x)^{\alpha-1},\quad x<g$
	$x=0$ pair 524.2 $\alpha=1$ pair 604 $\rho=0$ pair 606.1 $1\leqq R(\alpha)$ $R(\rho)=0$	
606.1	$\dfrac{e^{-xp}}{p^\alpha}$	$\dfrac{1}{\Gamma(\alpha)}(g-x)^{\alpha-1}\quad x<g$
	$x=0$ pair 521 $1\leqq R(\alpha)$	
607.0 Key	$\dfrac{1}{\cos^\alpha[\gamma(p+\lambda)]}$	$\dfrac{2^{\alpha-2}e^{-\lambda g}}{\pi\gamma\Gamma(\alpha)}\Gamma\left(\dfrac{\alpha\gamma+ig}{2\gamma}\right)\Gamma\left(\dfrac{\alpha\gamma-ig}{2\gamma}\right)$
	$\alpha=1, 2$ pairs 607.1, 607.8 $R(\alpha\gamma)\leqq 0$ $R(\frac{1}{2}\pi/\gamma)<\lvert R(\lambda)\rvert$ $1\leqq R(\alpha)$ $R(\frac{1}{2}\pi/\gamma)=\lvert R(\lambda)\rvert$	
607.1	$\dfrac{1}{\cos[\gamma(p+\lambda)]}$	$\dfrac{1}{2\gamma}e^{-\lambda g}\operatorname{sech}\left(\dfrac{\pi g}{2\gamma}\right)$
	$\lambda=0$ pair 609.0 $R(\frac{1}{2}\pi/\gamma)\leqq\lvert R(\lambda)\rvert$	
607.4	$\dfrac{p}{\cos\alpha p}$	$-\dfrac{\pi}{4\alpha^2}\tanh\left(\dfrac{\pi g}{2\alpha}\right)\operatorname{sech}\left(\dfrac{\pi g}{2\alpha}\right)$

No.	Coefficient $F(f)$	Coefficient $G(g)$
607.5	$\dfrac{p+\lambda}{\cos[\gamma(p+\lambda)]}$ $\boxed{S}$ $\lambda = 0$: pair 607.4 $\boxed{D}$ $R(\frac{1}{2}\pi/\gamma) \leqq \lvert R(\lambda)\rvert$	$-\dfrac{\pi}{4\gamma^2} e^{-\lambda g} \dfrac{\sinh\left(\dfrac{\pi g}{2\gamma}\right)}{\cosh^2\left(\dfrac{\pi g}{2\gamma}\right)}$
607.8	$\dfrac{1}{\cos^2[\gamma(p+\lambda)]}$ $\boxed{D}$ $R(\frac{1}{2}\pi/\gamma) \leqq \lvert R(\lambda)\rvert$	$\dfrac{g}{2\gamma^2} e^{-\lambda g} \operatorname{csch}\left(\dfrac{\pi g}{2\gamma}\right)$
608.0 Key	$\dfrac{1}{p+\lambda}\left\{\dfrac{1}{\sin[\gamma(p+\lambda)]} - \dfrac{1}{\gamma(p+\lambda)}\right\}$ $\boxed{S}$ $\lambda = 0$: pair 608.2 $\boxed{D}$ $R(\pi/\gamma) \leqq \lvert R(\lambda)\rvert$	$\dfrac{1}{\pi} e^{-\lambda g} \log\left[1 + \exp\left(-\dfrac{\pi}{\gamma}\lvert g\rvert\right)\right]$
608.2	$\dfrac{1}{p}\left(\dfrac{1}{\sin \alpha p} - \dfrac{1}{\alpha p}\right)$	$\dfrac{1}{\pi} \log\left[1 + \exp\left(-\dfrac{\pi\lvert g\rvert}{\alpha}\right)\right]$
609.0	$\dfrac{1}{\cos \alpha p}$ $\boxed{S}$ $\alpha = \frac{1}{2}$: pair 625	$\dfrac{1}{2\alpha} \operatorname{sech}\left(\dfrac{\pi g}{2\alpha}\right)$
610.0	$\dfrac{1}{p \cos \alpha p} - \dfrac{1}{p}$	$-\dfrac{1}{\pi} \operatorname{ctn}^{-1}\left[\sinh\left(\dfrac{\pi g}{2\alpha}\right)\right]$
610.1	$\dfrac{1}{p+\lambda}\left\{\dfrac{1}{\cos[\gamma(p+\lambda)]} - 1\right\}$ $\boxed{S}$ $\lambda = 0$: pair 610.0 $\boxed{D}$ $R(\frac{1}{2}\pi/\gamma) \leqq \lvert R(\lambda)\rvert$	$-\dfrac{1}{\pi} e^{-\lambda g} \operatorname{ctn}^{-1}\left[\sinh\left(\dfrac{\pi g}{2\gamma}\right)\right]$
611	$\dfrac{1}{\sin \alpha p} - \dfrac{1}{\alpha p}$	$\dfrac{1}{2\alpha}\left[\tanh\left(\dfrac{\pi g}{2\alpha}\right) \mp 1\right], \quad 0 < \pm g$
611.1	$\dfrac{1}{\sin[\gamma(p+\lambda)]} - \dfrac{1}{\gamma(p+\lambda)}$ $\boxed{S}$ $\lambda = 0$: pair 611 $\boxed{D}$ $R(\pi/\gamma) \leqq \lvert R(\lambda)\rvert$	$\dfrac{1}{2\gamma} e^{-\lambda g}\left[\tanh\left(\dfrac{\pi g}{2\gamma}\right) \mp 1\right], \quad 0 < \pm g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
612	$\tan \alpha p = \lim_{\beta \to 0} \left\{ \frac{\sin \alpha p}{\cos[(\alpha+\beta)p]} \right\}$	$-\frac{1}{2\alpha} \operatorname{csch}\left(\frac{\pi g}{2\alpha}\right)$	
612.1	$\frac{\tan \alpha p}{p}$	$\frac{1}{\pi} \log\left[\operatorname{ctnh}\left(\frac{\pi\|g\|}{4\alpha}\right)\right]$	
612.2	$\frac{\tan[\gamma(p+\lambda)]}{p+\lambda}$ $\boxed{S}\ \lambda = 0$ pair 612.1 $\boxed{B}\ R(\frac{1}{2}\pi/\gamma) \leqq \|R(\lambda)\|$	$\frac{1}{\pi} e^{-i\lambda g} \log\left[\operatorname{ctnh}\left(\frac{\pi\|g\|}{4\gamma}\right)\right]$	
612.3	$\tan \alpha p = \lim_{\beta \to 0} \left\{ \frac{\cos \alpha p}{\beta p \cos[(\alpha+\beta)p]} - \frac{1}{\beta p} \right\}$	$-\frac{1}{2\alpha} \operatorname{csch}\left(\frac{\pi g}{2\alpha}\right)$	
613	$\frac{1}{\tan \alpha p} - \frac{1}{\alpha p} = \lim_{\beta \to 0} \left\{ \frac{\cos \alpha p}{\sin[(\alpha+\beta)p]} - \frac{1}{(\alpha+\beta)p} \right\}$	$\frac{1}{2\alpha}\left[\operatorname{ctnh}\left(\frac{\pi g}{2\alpha}\right) \mp 1\right],$	$0 < \pm g$
613.1	$\frac{1}{p}\left(\frac{1}{\tan \alpha p} - \frac{1}{\alpha p}\right)$	$\frac{1}{\pi} \log\left[1 - \exp\left(-\frac{\pi\|g\|}{\alpha}\right)\right]$	
613.2	$\frac{1}{p+\lambda}\left\{ \frac{1}{\tan[\gamma(p+\lambda)]} - \frac{1}{\gamma(p+\lambda)} \right\}$ $\boxed{S}\ \lambda = 0$ pair 613.1 $\boxed{B}\ R(\pi/\gamma) \leqq \|R(\lambda)\|$	$\frac{1}{\pi} e^{-i\lambda g} \log\left[1 - \exp\left(-\frac{\pi\|g\|}{\gamma}\right)\right]$	
613.3	$\frac{1}{\tan \alpha p} - \frac{1}{\alpha p} = \lim_{\beta \to 0} \left\{ \frac{\alpha}{\beta(\alpha+\beta)p} - \frac{\sin \alpha p}{\beta p \sin[(\alpha+\beta)p]} \right\}$	$\frac{1}{2\alpha}\left[\operatorname{ctnh}\left(\frac{\pi g}{2\alpha}\right) \mp 1\right]$	$0 < \pm g$
614	$\frac{p}{\sin \alpha p}$	$\frac{\pi}{4\alpha^2} \operatorname{sech}^2\left(\frac{\pi g}{2\alpha}\right)$	

No.	Coefficient $F(f)$	Coefficient $G(g)$
614.1	$\dfrac{p+\lambda}{\sin[\gamma(p+\lambda)]}$ $\boxed{S}$ $\lambda = 0$: pair 614 ▣ $R(\pi/\gamma) \geqq \lvert R(\lambda)\rvert$	$\dfrac{\pi}{4\gamma^2} e^{-\lambda g} \operatorname{sech}^2\left(\dfrac{\pi g}{2\gamma}\right)$
615	$\dfrac{\sin \lambda p}{\sin \alpha p}$ $\boxed{S}$ $\lambda = \frac{1}{2}\alpha$: pair 609.0 $\boxed{S}$ $\lambda = 0$: pair 614 ▣ $R(\alpha) \geqq \lvert R(\lambda)\rvert$	$\dfrac{\dfrac{1}{2\alpha}\sin\left(\dfrac{\pi\lambda}{\alpha}\right)}{\cosh\left(\dfrac{\pi g}{\alpha}\right) + \cos\left(\dfrac{\pi\lambda}{\alpha}\right)}$
616	$\dfrac{\cos \lambda p}{\cos \alpha p}$ $\boxed{S}$ $\lambda = 0$: pair 609.0 ▣ $R(\alpha) \geqq \lvert R(\lambda)\rvert$	$\dfrac{\dfrac{1}{\alpha}\cos\left(\dfrac{\pi\lambda}{2\alpha}\right)\cosh\left(\dfrac{\pi g}{2\alpha}\right)}{\cosh\left(\dfrac{\pi g}{\alpha}\right) + \cos\left(\dfrac{\pi\lambda}{\alpha}\right)}$
616.4	$\dfrac{\sin \lambda p}{\cos \alpha p}$ $\boxed{S}$ $\lambda = 0$: pair 607.4 $\boxed{L}$ $\lambda \to \alpha$: pair 612 ▣ $R(\alpha) \geqq \lvert R(\lambda)\rvert$	$\dfrac{-\dfrac{1}{\alpha}\sin\left(\dfrac{\pi\lambda}{2\alpha}\right)\sinh\left(\dfrac{\pi g}{2\alpha}\right)}{\cosh\left(\dfrac{\pi g}{\alpha}\right) + \cos\left(\dfrac{\pi\lambda}{\alpha}\right)}$
616.7	$\dfrac{\cos \lambda p}{\sin \alpha p} - \dfrac{1}{\alpha p}$ $\boxed{S}$ $\lambda = \frac{1}{2}\alpha$: pair 611 $\boxed{S}$ $\lambda = 0$: pair 611 $\boxed{L}$ $\lambda \to \alpha$: pair 613 ▣ $R(\alpha) \geqq \lvert R(\lambda)\rvert$	$\dfrac{\dfrac{1}{2\alpha}\sinh\left(\dfrac{\pi g}{\alpha}\right)}{\cosh\left(\dfrac{\pi g}{\alpha}\right) + \cos\left(\dfrac{\pi\lambda}{\alpha}\right)} \mp \dfrac{1}{2\alpha}$, $0 < \pm g$
617	$\dfrac{\sin \lambda p}{p \sin \alpha p} - \dfrac{\lambda}{\alpha p}$ $\boxed{S}$ $\lambda = \frac{1}{2}\alpha$: pair 610.0 $\boxed{S}$ $\lambda = 0$: pair 611 $\boxed{L}$ $\lambda \to \alpha$: pair 613.3 ▣ $\lambda = \alpha$ ▣ $R(\alpha) < \lvert R(\lambda)\rvert$	$\dfrac{1}{\pi}\tan^{-1}\left[\tan\left(\dfrac{\pi\lambda}{2\alpha}\right)\tanh\left(\dfrac{\pi g}{2\alpha}\right)\right] \mp \dfrac{\lambda}{2\alpha}$, $0 < \pm g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
617 4	$\frac{1}{p}\left(\frac{\cos\lambda p}{\sin\alpha p}-\frac{1}{\alpha p}\right)$ $\lambda = \frac{1}{2}\alpha$ pair 608 2 $\lambda = \alpha$ pair 613 1 $\lambda = 0$ pair 608 2 $R(\alpha) < \lvert R(\lambda)\rvert$	$\frac{1}{2\pi}\log\left[2\cosh\left(\frac{\pi g}{\alpha}\right)+2\cos\left(\frac{\pi\lambda}{\alpha}\right)\right] - \frac{\lvert g\rvert}{2\alpha}$
617 7	$\frac{\sin\lambda p}{p\cos\alpha p}$ $\lambda = \alpha$ pair 612 1 $\lambda = 0$ pair 609 0 $R(\alpha) < \lvert R(\lambda)\rvert$	$\frac{1}{\pi}\tanh^{-1}\left[\sin\left(\frac{\pi\lambda}{2\alpha}\right)\operatorname{sech}\left(\frac{\pi g}{2\alpha}\right)\right]$
618	$\frac{\cos\lambda p}{p\cos\alpha p}-\frac{1}{p}$ $\lambda = 0$ pair 610 0 $\lambda \to \alpha$ pair 612.3 $\lambda = \alpha$ $R(\alpha) < \lvert R(\lambda)\rvert$	$-\frac{1}{\pi}\tan^{-1}\left[\cos\left(\frac{\pi\lambda}{2\alpha}\right)\operatorname{csch}\left(\frac{\pi g}{2\alpha}\right)\right]$
619	$\cosh xp = \lim_{\beta\to 0}\left[\frac{1}{2}(e^{xp}+e^{-xp})e^{\beta p}\right]$ $x = 0$ pair 403 1	$\frac{1}{2}[\mathfrak{S}_0(g+x)+\mathfrak{S}_0(g-x)]$
619 5	$\sinh xp = \lim_{\beta\to 0}\left[\frac{1}{2}(e^{xp}-e^{-xp})e^{-\beta p}\right]$ $x = 0$	$\frac{1}{2}[\mathfrak{S}_0(g+x)-\mathfrak{S}_0(g-x)]$
620	$\frac{\cosh(ap)}{p}-\frac{1}{p}$	$\mp\frac{1}{2}$ $\quad 0 < \pm g < a$
620 1	$\frac{\cosh[a(p+\lambda)]}{p+\lambda}-\frac{1}{p+\lambda}$ $\lambda = 0$ pair 620	$\mp\frac{1}{2}e^{-\lambda g}$ $\quad 0 < \pm g < a$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
621.4	$\dfrac{\cosh(ap)}{p+\lambda} - \dfrac{\cosh(a\lambda)}{p+\lambda}$ $\boxed{S}$ $\lambda = 0$: pair 620	$\mp \frac{1}{2}e^{-\lambda(g \mp a)}$,	$0 < \pm g < a$
622	$\dfrac{\sinh(ap)}{p}$	$\frac{1}{2}$,	$\lvert g\rvert < a$
622.1	$\dfrac{\sinh[a(p+\lambda)]}{p+\lambda}$ $\boxed{S}$ $\lambda = 0$: pair 622	$\frac{1}{2}e^{-\lambda g}$,	$\lvert g\rvert < a$
623	$\dfrac{\sinh(ap)}{p^2} - \dfrac{a}{p}$	$\frac{1}{2}(g \mp a)$,	$0 < \pm g < a$
623.1	$\dfrac{\sinh[a(p+\lambda)]}{(p+\lambda)^2} - \dfrac{a}{p+\lambda}$ $\boxed{S}$ $\lambda = 0$: pair 623	$\frac{1}{2}e^{-\lambda g}(g \mp a)$,	$0 < \pm g < a$
624.2	$\dfrac{\sinh(ap)}{p(p+\lambda)} - \dfrac{\sinh(a\lambda)}{\lambda(p+\lambda)}$ $\boxed{S}$ $\lambda = 0$: pair 623	$\dfrac{1}{2\lambda}[1 - e^{-\lambda(g \mp a)}]$,	$0 < \pm g < a$
624.5	$\dfrac{\sinh(ap)}{p^2} - \dfrac{a\cosh(ap)}{p}$	$\frac{1}{2}g$,	$\lvert g\rvert < a$
624.6	$\dfrac{\sinh[a(p+\lambda)]}{(p+\lambda)^2} - \dfrac{a\cosh[a(p+\lambda)]}{p+\lambda}$ $\boxed{S}$ $\lambda = 0$: pair 624.5	$\frac{1}{2}ge^{-\lambda g}$,	$\lvert g\rvert < a$
625	$\operatorname{sech} \pi f$	$\operatorname{sech} \pi g$	
631	$p^{\alpha-1}e^{-\rho\lvert p\rvert}$ $\boxed{S}$ $\alpha = n + \frac{1}{2}$: pair 631.2 $\boxed{S}$ $\alpha = \frac{1}{2}, 1, \frac{3}{2}, 2$: pairs 632.3, 632, 632.5, 632.7 $\boxed{S}$ $\rho = 0$: pair 521 $\boxed{C}$ $1 \leqq R(\alpha)$, $R(\rho) = 0$	$\dfrac{\Gamma(\alpha)}{\pi(\rho^2+g^2)^{\frac{1}{2}\alpha}}\sin\left[\frac{1}{2}\pi\alpha + \alpha\tan^{-1}\left(\dfrac{g}{\rho}\right)\right]$ $= \dfrac{\Gamma(\alpha)}{i2\pi}\left[\dfrac{1}{(-g-i\rho)^\alpha} - \dfrac{1}{(-g+i\rho)^\alpha}\right]$	

No	Coefficient $F(f)$	Coefficient $G(g)$
631.1	$p^{\alpha-2}e^{-\rho p} - p^{\alpha-2}$ $\boxed{S}$ $\alpha = 1$ pair 633 $\boxed{E}$ $\rho = \infty$ pair 521 $\boxed{R}$ $\rho = 0$ pair 522.8 $\boxed{D}$ $2 \geqq R(\alpha)$	$\dfrac{\Gamma(\alpha-1)}{\pi(\rho^2+g^2)^{\frac12\alpha-\frac12}}$ $\times \sin\left\{(\alpha-1)\left[\frac12\pi + \tan^{-1}\left(\frac{g}{\rho}\right)\right]\right\}$ $-\dfrac{(\frac12 \pm \frac12)}{\Gamma(2-\alpha)g^{\alpha-1}} \quad 0 < \pm g$
631.2	$p^{n-\frac12}e^{-\beta p}$ $\boxed{S}$ $n = 1, 2$ pairs 632.5, 632.9 $\boxed{E}$ $n = 0$ pair 632.3 $\boxed{L}$ $\beta \to 0$ pair 502.1	$\dfrac{(-1)^n\Gamma(n+\frac12)}{\pi(\beta^2+g^2)^{\frac12n+\frac14}}$ $\times \cos\left\{(n+\frac12)\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{g}{\beta}\right)\right]\right\}$
631.5	$p^{\alpha-2}(e^{-\rho p} - e^{-\sigma p})$ $\boxed{S}$ $\alpha = 1, 2$ pairs 633.3, 632.1 $\boxed{S}$ $\rho = -\sigma$ pair 644.2 $\boxed{S}$ $\rho = 0$ or $\sigma = 0$ pair 631.1 $\boxed{E}$ $\rho = \infty$ or $\sigma = \infty$ pair 631 $\boxed{R}$ $\rho = \sigma$ pair 639.1 $\boxed{D}$ $2 \geqq R(\alpha)$, $R(\rho) = 0$ or $R(\sigma) = 0$	$\dfrac{\Gamma(\alpha-1)}{\pi(\rho^2+g^2)^{\frac12\alpha-\frac12}}$ $\times \sin\left\{(\alpha-1)\left[\frac12\pi + \tan^{-1}\left(\frac{g}{\rho}\right)\right]\right\}$ $-\dfrac{\Gamma(\alpha-1)}{\pi(\sigma^2+g^2)^{\frac12\alpha-\frac12}}$ $\times \sin\left\{(\alpha-1)\left[\frac12\pi + \tan^{-1}\left(\frac{g}{\sigma}\right)\right]\right\}$ $= \dfrac{\Gamma(\alpha-1)}{i2\pi}\left[\dfrac{1}{(-g-i\rho)^{\alpha-1}} - \dfrac{1}{(-g+i\rho)^{\alpha-1}} - \dfrac{1}{(-g-i\sigma)^{\alpha}} + \dfrac{1}{(-g+i\sigma)^{\alpha-1}}\right]$
632	$e^{-\alpha\|p\|}$	$\dfrac{\alpha}{\pi(g^2+\alpha^2)}$
632.1	$e^{-\beta\|p\|} - e^{-\gamma\|p\|}$ $\boxed{A}$ pair 632.11 $\boxed{S}$ $\beta = \gamma$ pair 635 $\boxed{E}$ $\beta = \infty$ or $\gamma = \infty$ pair 632	$\dfrac{\beta}{\pi(\beta^2+g^2)} - \dfrac{\gamma}{\pi(\gamma^2+g^2)}$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
632.11	$\sin(\lambda\|p\|)e^{-\beta\|p\|}$ $\boxed{A}$ pair 632.1 $\boxed{S}$ $\lambda = 0$: pair 635 $\boxed{R}$ $\beta = \infty$ and $\lambda = \infty$: pair 632 $\boxed{L}$ $\beta \to 0$: pair 645 $\boxed{\square}$ $R(\beta) \geqq \|I(\lambda)\|$	$\dfrac{\lambda(\beta^2+\lambda^2-g^2)}{\pi[\beta^2+(\lambda+g)^2][\beta^2+(\lambda-g)^2]}$
632.2	$e^{-\beta\|p\|-xp}$ $\boxed{S}$ $x = 0$: pair 632 $\boxed{L}$ $\beta \to 0$: pair 601	$\dfrac{\beta}{\pi[\beta^2+(g-x)^2]}$
632.3	$\dfrac{1}{p^{\frac{1}{2}}}e^{-\rho\|p\|}$ $\boxed{S}$ $\rho = 0$: pair 522	$\left[\dfrac{g+(\rho^2+g^2)^{\frac{1}{2}}}{2\pi(\rho^2+g^2)}\right]^{\frac{1}{2}}$
632.5	$p^{\frac{1}{2}}e^{-\beta\|p\|}$ $\boxed{L}$ $\beta \to 0$: pair 503	$\dfrac{[(\beta^2+g^2)^{\frac{1}{2}}-2g][(\beta^2+g^2)^{\frac{1}{2}}+g]^{\frac{1}{2}}}{2^{\frac{3}{2}}\pi^{\frac{1}{2}}(\beta^2+g^2)^{\frac{3}{2}}}$
632.7	$pe^{-\beta\|p\|}$	$-\dfrac{2\beta g}{\pi(\beta^2+g^2)^2}$
632.9	$p^{\frac{3}{2}}e^{-\beta\|p\|}$ $\boxed{L}$ $\beta \to 0$: pair 504	$-3[2g(\beta^2+g^2)^{\frac{1}{2}}+\beta^2-3g^2]$ $\times\dfrac{[(\beta^2+g^2)^{\frac{1}{2}}+g]^{\frac{1}{2}}}{2^{\frac{5}{2}}\pi^{\frac{1}{2}}(\beta^2+g^2)^{\frac{5}{2}}}$
633	$\dfrac{1}{p}e^{-\rho\|p\|}-\dfrac{1}{p}$ $\boxed{\square}$ $\rho = 0$	$-\dfrac{1}{\pi}\tan^{-1}\left(\dfrac{\rho}{g}\right)$
633.3	$\dfrac{1}{p}(e^{-\rho\|p\|}-e^{-\sigma\|p\|})$ $\boxed{A}$ pair 633.4 $\boxed{S}$ $\sigma = 0$ or $\rho = 0$: pair 633 $\boxed{R}$ $\rho = \sigma$: pair 638.1	$\dfrac{1}{\pi}\left[\tan^{-1}\left(\dfrac{\sigma}{g}\right)-\tan^{-1}\left(\dfrac{\rho}{g}\right)\right]$

TABLE 1 (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
633.4	$\frac{1}{p}\sin(\lambda\|p\|)e^{-\rho\|p\|}$ $\boxed{\text{A}}$ pair 633.3 $\boxed{\text{S}}$ $\rho = 0$ pair 644.4 $\boxed{\text{R}}$ $\lambda = 0$ pair 638.1 $\boxed{\text{C}}$ $R(\rho) < \|I(\lambda)\|$	$\frac{1}{4\pi}\log\left[\frac{\rho^2+(\lambda+g)^2}{\rho^2+(\lambda-g)^2}\right]$
634	$\frac{1}{\|p\|^{1/2}}e^{-\rho\|p\|}$ $\boxed{\text{S}}$ $\rho = 0$ pair 523.1	$\left[\frac{\rho+(\rho^2+g^2)^{1/2}}{2\pi(\rho^2+g^2)}\right]^{1/2}$
634.5	$\|p\|^{1/2}e^{-\beta\|p\|}$ $\boxed{\text{L}}$ $\beta \to 0$ pair 505.1	$\frac{[2\beta-(\beta^2+g^2)^{1/2}][\beta+(\beta^2+g^2)^{1/2}]^{1/2}}{2^{3/2}\pi^{1/2}(\beta^2+g^2)^{3/2}}$
635	$\|p\|e^{-\beta\|p\|}$	$\frac{\beta^2-g^2}{\pi(\beta^2+g^2)^2}$
636.0	$\|p\|^{\alpha-1}e^{-\rho\|p\|}$ $\boxed{\text{S}}$ $\alpha = \frac{1}{2}, 1, \frac{3}{2}, 2$ pairs 634, 632, 634.5, 635 $\boxed{\text{S}}$ $\rho = 0$ pair 522.5 $\boxed{\text{C}}$ $1 \leqq R(\alpha),\ R(\rho) = 0$	$\frac{\Gamma(\alpha)}{\pi(\rho^2+g^2)^{\alpha/2}}\cos\left[\alpha\tan^{-1}\left(\frac{g}{\rho}\right)\right]$ $= \frac{\Gamma(\alpha)}{2\pi}\left[\frac{1}{(\rho+ig)^{\alpha}}+\frac{1}{(\rho-ig)^{\alpha}}\right]$
638.1	$\frac{\|p\|}{p}e^{-\beta\|p\|}$	$\frac{g}{\pi(\beta^2+g^2)}$
639.1	$\|p\|p^{\alpha-2}e^{-\rho\|p\|}$ $\boxed{\text{S}}$ $\alpha = 1, 2$ pairs 638.1, 635 $\boxed{\text{S}}$ $\rho = 0$ pair 522.8 $\boxed{\text{C}}$ $1 \leqq R(\alpha),\ R(\rho) = 0$	$-\frac{\Gamma(\alpha)}{\pi(\rho^2+g^2)^{\alpha/2}}\cos\left[\frac{1}{2}\pi\alpha+\alpha\tan^{-1}\left(\frac{g}{\rho}\right)\right]$ $= -\frac{\Gamma(\alpha)}{2\pi}\left[\frac{1}{(-g-i\rho)^{\alpha}}+\frac{1}{(-g+i\rho)^{\alpha}}\right]$
640.1	$\frac{\|p\|^{1/2}}{p}e^{-\rho\|p\|}$ $\boxed{\text{S}}$ $\rho = 0$ pair 523.5	$\frac{g}{\{2\pi(\rho^2+g^2)[\rho+(\rho^2+g^2)^{1/2}]\}^{1/2}}$
641.1	$\frac{p}{\|p\|^{1/2}}e^{-\beta\|p\|}$	$-\frac{g[2\beta+(\beta^2+g^2)^{1/2}]}{2^{3/2}\pi^{1/2}(\beta^2+g^2)^{3/2}[\beta+(\beta^2+g^2)^{1/2}]^{1/2}}$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
644.2	$p^{\alpha-2}\sin(x\lvert p\rvert)$ $\boxed{S}$ $\alpha = 1$: pair 644.4 $\boxed{R}$ $x = 0$: pair 522.8 $\boxed{C}$ $2 \leqq R(\alpha)$	$\begin{cases} \dfrac{\Gamma(\alpha-1)}{2\pi}\left[\dfrac{1}{(-g+x)^{\alpha-1}} - \dfrac{1}{(-g-x)^{\alpha-1}}\right], & g < -\lvert x\rvert \\ \dfrac{x\Gamma(\alpha-1)}{2\pi\lvert x\rvert}\left[\dfrac{1}{(-g+\lvert x\rvert)^{\alpha-1}} + \dfrac{\cos\pi\alpha}{(g+\lvert x\rvert)^{\alpha-1}}\right], & \lvert g\rvert < \lvert x\rvert \\ \dfrac{\cos\pi\alpha\,\Gamma(\alpha-1)}{2\pi}\left[\dfrac{1}{(g+x)^{\alpha-1}} - \dfrac{1}{(g-x)^{\alpha-1}}\right], & \lvert x\rvert < g \end{cases}$
644.4	$\dfrac{1}{p}\sin(x\lvert p\rvert)$ $\boxed{C}$ $x = 0$	$\dfrac{1}{2\pi}\log\left\lvert\dfrac{g+x}{g-x}\right\rvert$
645	$\sin(x\lvert p\rvert) = \lim\limits_{\beta\to 0}\,[\sin(x\lvert p\rvert)e^{-\beta\lvert p\rvert}]$ $\boxed{S}$ $x = 0$: pair 407	$\dfrac{x}{\pi(x^2-g^2)}$
650.0	$\dfrac{1}{(p+\rho)^{\alpha}}\exp\left[\dfrac{1}{\lambda(p+\rho)}\right]$ $\boxed{S}$ $\alpha = \frac{1}{2}, 1, \frac{3}{2}, 2$: pairs 651, 655.1, 653, 656.1 $\boxed{S}$ $\lambda = 1$: pair 650.5 $\boxed{S}$ $\rho = 0$: pair 650.4 $\boxed{R}$ $\lambda = \dot{\infty}$: pair 524.2 $\boxed{C}$ $2 \leqq R(\alpha)[\frac{1}{2} \leqq R(\alpha)$ for transposed pair$]$, $R(\rho) = 0$ $\boxed{C}$ $\lambda = 0$ $\boxed{C}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$	$(\lambda g)^{\frac{1}{2}\alpha-\frac{1}{2}}e^{-\rho g}I_{\alpha-1}\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$, $\quad 0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
650 4	$\frac{1}{p^{\alpha}}\exp\left(-\frac{1}{cp}\right)$ S $\alpha = \frac{1}{2}, 1, \frac{3}{2}$ pairs 652 2, 655 2, 653 4 S $c = \infty$ pair 521 I $2 \leqq R(\alpha)$ [$\frac{1}{2} \leqq R(\alpha)$ for transposed pair]	$(cg)^{\frac{1}{2}(\alpha-1)} J_{\alpha-1}\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$0 < g$
650 5	$\frac{1}{(p+\beta)^{\alpha}}\exp\left(\frac{1}{p+\beta}\right)$ S $\alpha = \frac{1}{2}, 1, \frac{3}{2}, 2$ pairs 651 5, 655 5, 653 5, 656 5	$g^{\frac{1}{2}(\alpha-1)} e^{-\beta g} I_{\alpha-1}(2g^{\frac{1}{2}})$,	$0 < g$
651	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp\left[\frac{1}{\lambda(p+\rho)}\right]$ S $\lambda = 1$ pair 651 5 S $\rho = 0$ pair 652 2 S $\lambda = \infty$ pair 526 S $\lambda = 0$ I $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$	$\frac{1}{(\pi g)^{\frac{1}{2}}} e^{-\rho g}\cosh\left(\frac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
651 5	$\frac{1}{(p+\beta)^{\frac{1}{2}}}\exp\left(\frac{1}{p+\beta}\right)$ L $\beta \to 0$ pair 652	$\frac{1}{(\pi g)^{\frac{1}{2}}} e^{-\beta g}\cosh(2g^{\frac{1}{2}})$,	$0 < g$
652	$\frac{1}{p^{\frac{1}{2}}}\exp\left(\frac{1}{p}\right)$ $= \lim_{\beta\to 0}\left[\frac{1}{(p+\beta)^{\frac{1}{2}}}\exp\left(\frac{1}{p+\beta}\right)\right]$ F This F coefficient has a regular mate given by pair 660 1	$\frac{1}{(\pi g)^{\frac{1}{2}}}\cosh(2g^{\frac{1}{2}})$,	$0 < g$
652 2	$\frac{1}{p^{\frac{1}{2}}}\exp\left(-\frac{1}{cp}\right)$ S $c = \infty$ pair 522	$\frac{1}{(\pi g)^{\frac{1}{2}}}\cos\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$	
653	$\dfrac{1}{(p+\rho)^{3/2}}\exp\left[\dfrac{1}{\lambda(p+\rho)}\right]$	$\dfrac{\lambda^{1/2}}{\pi^{1/2}}e^{-\rho g}\sinh\left(\dfrac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
	$\boxed{S}$ $\lambda = 1$: pair 653.5 $\boxed{S}$ $\rho = 0$: pair 653.4 $\boxed{B}$ $\lambda = \infty$: pair 529 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$ $\boxed{D}$ $R(\rho) = 0$ for transposed pair		
653.4	$\dfrac{1}{p^{3/2}}\exp\left(-\dfrac{1}{cp}\right)$	$\dfrac{c^{1/2}}{\pi^{1/2}}\sin\left(\dfrac{2g^{1/2}}{c^{1/2}}\right)$,	$0 < g$
	$\boxed{E}$ No transposed pair		
653.5	$\dfrac{1}{(p+\beta)^{3/2}}\exp\left(\dfrac{1}{p+\beta}\right)$	$\dfrac{1}{\pi^{1/2}}e^{-\beta g}\sinh(2g^{1/2})$,	$0 < g$
654.2	$\exp\left[\dfrac{1}{\lambda(p+\rho)}\right] - 1$	$\dfrac{1}{(\lambda g)^{1/2}}e^{-\rho g}I_1\left(\dfrac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
	$\boxed{S}$ $\rho = 0$: pair 654.3 $\boxed{B}$ $\lambda = \infty$: pair 438 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$		
654.3	$\exp\left(-\dfrac{1}{cp}\right) - 1$	$-\dfrac{1}{(cg)^{1/2}}J_1\left(\dfrac{2g^{1/2}}{c^{1/2}}\right)$,	$0 < g$
654.6	$\exp\left(\dfrac{1}{cp}\right) - 1$	$-\dfrac{1}{(cg)^{1/2}}I_1\left(\dfrac{2g^{1/2}}{c^{1/2}}\right)$,	$g < 0$
655.1	$\dfrac{1}{p+\rho}\exp\left(\dfrac{1}{\lambda(p+\rho)}\right)$	$e^{-\rho g}I_0\left(\dfrac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
	$\boxed{S}$ $\lambda = 1$: pair 655.5 $\boxed{S}$ $\rho = 0$: pair 655.2 $\boxed{B}$ $\lambda = \infty$: pair 438 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$		

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
655.2	$\frac{1}{p}\exp\left(-\frac{1}{cp}\right)$	$J_0\left(\frac{2g^{1/2}}{c^{1/2}}\right)$,	$0 < g$
655.3	$\frac{1}{p}\exp\left(\frac{1}{cp}\right)$	$-I_0\left(\frac{2g^{1/2}}{c^{1/2}}\right)$,	$g < 0$
655.5	$\frac{1}{p+\beta}\exp\left(\frac{1}{p+\beta}\right)$	$e^{-\beta g}I_0(2g^{1/2})$,	$0 < g$
656.1	$\frac{1}{(p+\beta)^2}\exp\left[\frac{1}{\lambda(p+\beta)}\right]$ $\boxed{A}$ pair 656.2 $\boxed{S}$ $\lambda = 1$ pair 656.5 $\boxed{B}$ $\lambda = \infty$ pair 442 $\boxed{D}$ $\lambda = 0$	$(\lambda g)^{1/2}e^{-\beta g}I_1\left(\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
656.2	$\frac{1}{(p+\beta)^2}\exp\left[-\frac{1}{\lambda(p+\beta)}\right]$ $\boxed{A}$ pair 656.1 $\boxed{S}$ $\lambda = -1$ pair 656.5 $\boxed{B}$ $\lambda = \infty$ pair 442 $\boxed{L}$ $\beta \to 0$ pair 656.4 $\boxed{D}$ $\lambda = 0$	$(\lambda g)^{1/2}e^{-\beta g}J_1\left(\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
656.4	$\frac{1}{p^2}\exp\left(-\frac{1}{\lambda p}\right)$ $= \lim_{\beta\to 0}\left\{\frac{1}{(p+\beta)^2}\exp\left[-\frac{1}{\lambda(p+\beta)}\right]\right\}$ $\boxed{B}$ $\lambda = \infty$ pair 408.1 with $k = 2$ $\boxed{D}$ $\lambda = 0$	$(\lambda g)^{1/2}J_1\left(\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
656.5	$\frac{1}{(p+\beta)^2}\exp\left(\frac{1}{p+\beta}\right)$	$g^{1/2}e^{-\beta g}I_1(2g^{1/2})$,	$0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
660.1	$\frac{1}{p^{\frac12}}\exp\left(\frac{1}{cp}\right)$ $\boxed{B}$ $c = \infty$: pair 522	$\begin{cases} -\frac{1}{(\pi g)^{\frac12}}\sinh\left(\frac{2g^{\frac12}}{c^{\frac12}}\right), & g<0 \\ \frac{1}{(\pi g)^{\frac12}}\left[\cosh\left(\frac{2g^{\frac12}}{c^{\frac12}}\right)-\sinh\left(\frac{2g^{\frac12}}{c^{\frac12}}\right)\right] & \\ \quad = \frac{1}{(\pi g)^{\frac12}}\exp\left(-\frac{2g^{\frac12}}{c^{\frac12}}\right), & 0<g \end{cases}$
661.2	$\frac{1}{p^{\alpha}}\exp\left(\frac{1}{cp}\right)$ $\boxed{S}$ $\alpha = \frac12, 1, \frac32$: pairs 660.1, 655.3, 662.1 $\boxed{B}$ $c = \infty$: pair 521 $\boxed{O}$ $2 \leqq R(\alpha)$ [$\frac32 \leqq R(\alpha)$ for transposed pair]	$\begin{cases} -(cg)^{\frac12\alpha-\frac12}I_{1-\alpha}\left(\frac{2g^{\frac12}}{c^{\frac12}}\right), & g<0 \\ (cg)^{\frac12\alpha-\frac12}\left[I_{\alpha-1}\left(\frac{2g^{\frac12}}{c^{\frac12}}\right)-I_{1-\alpha}\left(\frac{2g^{\frac12}}{c^{\frac12}}\right)\right] & \\ \quad = \frac{2\sin\pi\alpha}{\pi}(cg)^{\frac12\alpha-\frac12}K_{\alpha-1}\left(\frac{2g^{\frac12}}{c^{\frac12}}\right), & 0<g \end{cases}$
662.1	$\frac{1}{p^{\frac32}}\exp\left(\frac{1}{cp}\right)$ $\boxed{E}$ No transposed pair	$\begin{cases} -\frac{c^{\frac12}}{\pi^{\frac12}}\cosh\left(\frac{2g^{\frac12}}{c^{\frac12}}\right), & g<0 \\ \frac{c^{\frac12}}{\pi^{\frac12}}\left[\sinh\left(\frac{2g^{\frac12}}{c^{\frac12}}\right)-\cosh\left(\frac{2g^{\frac12}}{c^{\frac12}}\right)\right] & \\ \quad = -\frac{c^{\frac12}}{\pi^{\frac12}}\exp\left(-\frac{2g^{\frac12}}{c^{\frac12}}\right), & 0<g \end{cases}$

Part 7. Exponential and Trigonometric Functions of f^2

702	$\phi_n(f) = e^{\pi f^2}D_f^n e^{-2\pi f^2}$ $\quad = (-)^n e^{-\frac12 x^2}(4\pi)^{\frac12 n}H_n(x)$ $\boxed{N}$ $x = (4\pi)^{\frac12}f$ $\boxed{S}$ $n = 0, 1, \cdots, 9$: pairs 704.0, 704.1, $\cdots$, 704.9	$i^n\phi_n(g)$
704.0	$\phi_0(f) = e^{-\frac12 x^2}$ $\boxed{N}$ $x = (4\pi)^{\frac12}f$ $\boxed{A}$ pair 705.1	$\phi_0(g)$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
704.1	$\phi_1(f) = -e^{-\frac{1}{2}x^2}(4\pi)^{\frac{1}{2}}x$ $x = (4\pi)^{\frac{1}{2}}f$	$i\phi_1(g)$
704.2	$\phi_2(f) = e^{-\frac{1}{2}x^2}(4\pi)(x^2 - 1)$ $x = (4\pi)^{\frac{1}{2}}f$	$-\phi_2(g)$
704.3	$\phi_3(f) = -e^{-\frac{1}{2}x^2}(4\pi)^{\frac{3}{2}}(x^3 - 3x)$ $x = (4\pi)^{\frac{1}{2}}f$	$-i\phi_3(g)$
704.4	$\phi_4(f) = e^{-\frac{1}{2}x^2}(4\pi)^2(x^4 - 6x^2 + 3)$ $x = (4\pi)^{\frac{1}{2}}f$	$\phi_4(g)$
704.5	$\phi_5(f) = -e^{-\frac{1}{2}x^2}(4\pi)^{\frac{5}{2}}(x^5 - 10x^3 + 15x)$ $x = (4\pi)^{\frac{1}{2}}f$	$i\phi_5(g)$
704.6	$\phi_6(f) = e^{-\frac{1}{2}x^2}(4\pi)^3(x^6 - 15x^4 + 45x^2 - 15)$ $x = (4\pi)^{\frac{1}{2}}f$	$-\phi_6(g)$
704.7	$\phi_7(f) = -e^{-\frac{1}{2}x^2}(4\pi)^{\frac{7}{2}}(x^7 - 21x^5 + 105x^3 - 105x)$ $x = (4\pi)^{\frac{1}{2}}f$	$-i\phi_7(g)$
704.8	$\phi_8(f) = e^{-\frac{1}{2}x^2}(4\pi)^4(x^8 - 28x^6 + 210x^4 - 420x^2 + 105)$ $x = (4\pi)^{\frac{1}{2}}f$	$\phi_8(g)$
704.9	$\phi_9(f) = -e^{-\frac{1}{2}x^2}(4\pi)^{\frac{9}{2}}(x^9 - 36x^7 + 378x^5 - 1260x^3 + 945x)$ $x = (4\pi)^{\frac{1}{2}}f$	$i\phi_9(g)$
705.1	$\exp(-\pi f^2)$ pair 704.0	$\exp(-\pi g^2)$

No.	Coefficient $F(f)$	Coefficient $G(g)$
706.1	$(x^2-3)^2\exp(-\frac{1}{4}x^2)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$(y^2-3)^2\exp(-\frac{1}{4}y^2)$ $\boxed{N}\ y=(4\pi)^{\frac12}g$
706.2	$(x^4-14x^2+21)^2\exp(-\frac{1}{4}x^2)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$(y^4-14y^2+21)^2\exp(-\frac{1}{4}y^2)$ $\boxed{N}\ y=(4\pi)^{\frac12}g$
706.3	$(x^6-33x^4+231x^2-231)^2\exp(-\frac{1}{4}x^2)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$(y^6-33y^4+231y^2-231)^2\exp(-\frac{1}{4}y^2)$ $\boxed{N}\ y=(4\pi)^{\frac12}g$
706.4	$(x^6-33x^4+171x^2-531)^2\exp(-\frac{1}{4}x^2)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$(y^6-33y^4+171y^2-531)^2\exp(-\frac{1}{4}y^2)$ $\boxed{N}\ y=(4\pi)^{\frac12}g$
706.5	$(x^8-60x^6+1110x^4-5340x^2+5265)^2 \times \exp(-\frac{1}{4}x^2)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$(y^8-60y^6+1110y^4-5340y^2+5265)^2 \times \exp(-\frac{1}{4}y^2)$ $\boxed{N}\ y=(4\pi)^{\frac12}g$
706.6	$(x^8-60x^6+990x^4-4620x^2+3465)^2 \times \exp(-\frac{1}{4}x^2)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$(y^8-60y^6+990y^4-4620y^2+3465)^2 \times \exp(-\frac{1}{4}y^2)$ $\boxed{N}\ y=(4\pi)^{\frac12}g$
706.7	$\chi_n(f)=\exp(-\pi f^2) \times {}_1F_1(-n;\frac{3}{4}-\frac{1}{2}n;\pi f^2)$ $\boxed{S}\ n=0,1$: pairs 704.0, 704.2 $\boxed{S}\ n=2,3,4$: pairs 706.72, 706.73, 706.74	$(-1)^n\chi_n(g)$
706.72	$\chi_2(f)=-\frac{1}{3}\exp(-\frac{1}{4}x^2)(x^4-6x^2-3)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$\chi_2(g)$
706.73	$\chi_3(f)=\frac{1}{15}\exp(-\frac{1}{4}x^2) \times (x^6-15x^4+15x^2+15)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$-\chi_3(g)$
706.74	$\chi_4(f)=\frac{1}{105}\exp(-\frac{1}{4}x^2) \times (x^8-28x^6+126x^4+84x^2+105)$ $\boxed{N}\ x=(4\pi)^{\frac12}f$	$\chi_4(g)$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
707.0	$p^n \exp(-\pi\beta f^2)$ $\boxed{A}$ pair 709.0 $\boxed{S}$ $n = 1, 2, 3, 4$, pairs 708.1, 708.2, 708.3, 708.4 $\boxed{S}$ $\beta = \frac{1}{2}$, pair 720.0 $\boxed{S}$ $n = 0$, pair 708.0	$\frac{(-1)^n(2\pi)^{\frac{1}{2}n}}{\beta^{\frac{1}{2}(n+1)}} \exp\left(-\frac{\pi g^2}{\beta}\right) H_n\left(\frac{2^{\frac{1}{2}}\pi^{\frac{1}{2}} g}{\beta^{\frac{1}{2}}}\right)$ $= \frac{1}{2^{\frac{1}{2}n}\beta^{\frac{1}{2}(n+1)}} \exp\left(-\frac{\pi g^2}{2\beta}\right) \phi_n\left(\frac{g}{2^{\frac{1}{2}}\beta^{\frac{1}{2}}}\right)$ $= \frac{1}{\beta^{\frac{1}{2}}} D_s^n \exp\left(-\frac{\pi g^2}{\beta}\right)$
708.0	$\exp(-\pi\beta f^2)$ $\boxed{A}$ pair 710.0 $\boxed{S}$ $\beta = \pm i$, pair 760 $\boxed{C}$ $R(\beta) = 0 \neq \beta$	$\frac{1}{\beta^{\frac{1}{2}}} \exp\left(-\frac{\pi g^2}{\beta}\right)$
708.1	$p \exp(-\pi\beta f^2)$ $\boxed{A}$ pair 710.1 $\boxed{S}$ $\beta = \frac{1}{2}$, pair 721.1 $\boxed{L}$ $\beta \to 0$, pair 404.1	$-\frac{2\pi g}{\beta^{\frac{3}{2}}} \exp\left(-\frac{\pi g^2}{\beta}\right)$
708.2	$p^2 \exp(-\pi\beta f^2)$ $\boxed{A}$ pair 710.2 $\boxed{S}$ $\beta = \frac{1}{2}$, pair 721.2	$\frac{2\pi(2\pi g^2 - \beta)}{\beta^{\frac{5}{2}}} \exp\left(-\frac{\pi g^2}{\beta}\right)$
708.3	$p^3 \exp(-\pi\beta f^2)$ $\boxed{A}$ pair 710.3 $\boxed{S}$ $\beta = \frac{1}{2}$, pair 721.3	$-\frac{4\pi^2 g(2\pi g^2 - 3\beta)}{\beta^{\frac{7}{2}}} \exp\left(-\frac{\pi g^2}{\beta}\right)$
708.4	$p^4 \exp(-\pi\beta f^2)$ $\boxed{A}$ pair 710.4 $\boxed{S}$ $\beta = \frac{1}{2}$, pair 721.4	$\frac{4\pi^2(4\pi^2 g^4 - 12\pi\beta g^2 + 3\beta^2)}{\beta^{\frac{9}{2}}} \exp\left(-\frac{\pi g^2}{\beta}\right)$
709.0	$p^n \exp(\rho p^2)$ $\boxed{A}$ pair 707.0 $\boxed{S}$ $n = 1, 2, 3, 4$, pairs 710.1, 710.2, 710.3, 710.4 $\boxed{S}$ $\rho = 1/(8\pi)$, pair 720.0 $\boxed{S}$ $n = 0$, pair 710.0 $\boxed{C}$ $R(\rho) = 0$	$\frac{(-1)^n}{2^{n+1}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}(n+1)}} \exp\left(-\frac{g^2}{4\rho}\right) H_n\left(\frac{g}{2\rho^{\frac{1}{2}}}\right)$ $= \frac{1}{2^{(n+1)}(\pi\rho)^{\frac{1}{2}(n+1)}}$ $\times \exp\left(-\frac{g^2}{8\rho}\right) \phi_n\left(\frac{g}{2^{\frac{1}{2}}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}}}\right)$ $= \frac{1}{2(\pi\rho)^{\frac{1}{2}}} D_s^n \exp\left(-\frac{g^2}{4\rho}\right)$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
710.0	$\exp(\rho p^2)$ $\boxed{\text{A}}$ pair 708.0 $\boxed{\text{S}}$ $\rho = 1/(8\pi),\ 1/(4\pi),\ \pm i/(4\pi)$: pairs 721.0, 705.1, 760 $\boxed{\text{D}}$ $\rho = 0$	$\dfrac{1}{2\pi^{1/2}\rho^{1/2}}\exp\left(-\dfrac{g^2}{4\rho}\right)$
710.1	$p\exp(\alpha p^2)$ $\boxed{\text{A}}$ pair 708.1 $\boxed{\text{S}}$ $\alpha = 1/(8\pi)$: pair 721.1 $\boxed{\text{L}}$ $\alpha \to 0$: pair 404.1	$-\dfrac{g}{4\pi^{1/2}\alpha^{3/2}}\exp\left(-\dfrac{g^2}{4\alpha}\right)$
710.2	$p^2\exp(\alpha p^2)$ $\boxed{\text{A}}$ pair 708.2 $\boxed{\text{S}}$ $\alpha = 1/(8\pi)$: pair 721.2	$\dfrac{g^2-2\alpha}{8\pi^{1/2}\alpha^{5/2}}\exp\left(-\dfrac{g^2}{4\alpha}\right)$
710.3	$p^3\exp(\alpha p^2)$ $\boxed{\text{A}}$ pair 708.3 $\boxed{\text{S}}$ $\alpha = 1/(8\pi)$: pair 721.3	$-\dfrac{g^3-6\alpha g}{16\pi^{1/2}\alpha^{7/2}}\exp\left(-\dfrac{g^2}{4\alpha}\right)$
710.4	$p^4\exp(\alpha p^2)$ $\boxed{\text{A}}$ pair 708.4 $\boxed{\text{S}}$ $\alpha = 1/(8\pi)$: pair 721.4	$\dfrac{g^4-12\alpha g^2+12\alpha^2}{32\pi^{1/2}\alpha^{9/2}}\exp\left(-\dfrac{g^2}{4\alpha}\right)$
715.0	$\exp(\alpha p^2)\,{}_1F_1(-n;\tfrac{3}{4}-\tfrac{1}{2}n;-\alpha p^2)$ $\boxed{\text{S}}$ $\alpha = 1/(4\pi)$: pair 706.7 $\boxed{\text{B}}$ $n = 0$: pair 710.0	$\dfrac{(-1)^n}{2\pi^{1/2}\alpha^{1/2}}\exp\left(-\dfrac{g^2}{4\alpha}\right)$ $\times\,{}_1F_1\left(-n;\tfrac{3}{4}-\tfrac{1}{2}n;\dfrac{g^2}{4\alpha}\right)$
720.0	$p^n\exp(-\tfrac{1}{2}\pi f^2)$ $\boxed{\text{S}}$ $n = 0, 1, \cdots, 4$: pairs 721.0, 721.1, $\cdots$, 721.4	$(-1)^n 2^{n+\frac{1}{2}}\pi^{\frac{1}{2}n}\exp(-2\pi g^2)H_n(2\pi^{\frac{1}{2}}g)$ $= 2^{\frac{1}{2}}\exp(-\pi g^2)\phi_n(g)$ $= 2^{\frac{1}{2}}D_g{}^n\exp(-2\pi g^2)$
721.0	$\exp(-\tfrac{1}{2}\pi f^2)$	$2^{\frac{1}{2}}\exp(-2\pi g^2)$
721.1	$p\exp(-\tfrac{1}{2}\pi f^2)$	$-2^{\frac{5}{2}}\pi g\exp(-2\pi g^2)$

TABLE 1 (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
721 2	$p^2 \exp(-\frac{1}{2}\pi f^2)$	$2^{\frac{5}{2}}\pi(4\pi g^2 - 1)\exp(-2\pi g^2)$
721 3	$p^3 \exp(-\frac{1}{2}\pi f^2)$	$-2^{\frac{9}{2}}\pi^2 g(4\pi g^2 - 3)\exp(-2\pi g^2)$
721 4	$p^4 \exp(-\frac{1}{2}\pi f^2)$	$2^{\frac{9}{2}}\pi^2(16\pi^2 g^4 - 24\pi g^2 + 3)\exp(-2\pi g^2)$
725 1	$\frac{1}{p}\exp(-\pi\beta f^2) - \frac{1}{p}$ $\boxed{A}$ pair 727 $\boxed{C}$ $R(\beta) = 0 \dotplus \beta$	$\frac{1}{2}\operatorname{erf}\left(\frac{\pi^{\frac{1}{2}} g}{\beta^{\frac{1}{2}}}\right) \mp \frac{1}{2}$ $\qquad 0 < \pm g$
726 1	$\frac{1}{p^2}\exp(-\pi\beta f^2) - \frac{1}{p^2}$ $\boxed{A}$ pair 727 2 $\boxed{C}$ $R(\beta) = 0 \dotplus \beta$	$\frac{g}{2}\left[\operatorname{erf}\left(\frac{\pi^{\frac{1}{2}} g}{\beta^{\frac{1}{2}}}\right) + \frac{\beta^{\frac{1}{2}}}{\pi g}\exp\left(-\frac{\pi g^2}{\beta}\right) \mp 1\right],\quad 0 < \pm g$
726 2	$\frac{1}{p^k}\exp(-\pi\beta f^2)$ $\boxed{S}$ $k = 0$ pair 708 0 $\boxed{D}$ $\beta \to 0$ pair 410 $\boxed{C}$ $R(\beta) = 0 \dotplus \beta$ $\boxed{E}$ This pair is a formal pair only, obtained as the result of the k fold application of pair 210 to regular pair 708 0	$\frac{1}{\beta^{\frac{1}{2}}} D_0^{-k} \exp\left(-\frac{\pi g^2}{\beta}\right)$
727	$\frac{1}{p}\exp(\rho p^2) - \frac{1}{p}$ $\boxed{A}$ pair 725 1 $\boxed{D}$ $\rho = 0$	$\mp\frac{1}{2}\operatorname{erfc}\left(\frac{\lvert g\rvert}{2\rho^{\frac{1}{2}}}\right), \qquad 0 < \pm g$
727 2	$\frac{1}{p^2}\exp(\rho p^2) - \frac{1}{p^2}$ $\boxed{A}$ pair 726 1 $\boxed{D}$ $\rho = 0$	$\frac{\rho^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\exp\left(-\frac{g^2}{4\rho}\right) - \frac{1}{2}\lvert g\rvert\operatorname{erfc}\left(\frac{\lvert g\rvert}{2\rho^{\frac{1}{2}}}\right)$

No.	Coefficient $F(f)$	Coefficient $G(g)$
727.9	$\exp(\rho p^2) - \exp(\sigma p^2)$ $\boxed{\text{S}}$ $\rho = -\sigma$: pair 751 $\boxed{\text{B}}$ $\rho = \infty$ or $\sigma = \infty$: pair 710.0 $\boxed{\text{R}}$ $\rho = \sigma$: pair 710.2 $\boxed{\text{D}}$ $\rho = 0$ $\boxed{\text{D}}$ $\sigma = 0$	$\frac{1}{2\pi^{\frac12}\rho^{\frac12}}\exp\left(-\frac{g^2}{4\rho}\right) - \frac{1}{2\pi^{\frac12}\sigma^{\frac12}}\exp\left(-\frac{g^2}{4\sigma}\right)$
728.1	$\frac{\exp(\rho p^2)}{p+\lambda} - \frac{\exp(\rho\lambda^2)}{p+\lambda}$ $\boxed{\text{S}}$ $\lambda = 0$: pair 727 $\boxed{\text{B}}$ $\rho = \infty$: pairs 438 and 439 $\boxed{\text{D}}$ $\rho = 0$	$\frac12\exp(\rho\lambda^2 - \lambda g)\left[\operatorname{erf}\left(\frac{g}{2\rho^{\frac12}} - \lambda\rho^{\frac12}\right) \mp 1\right], \quad 0 < \pm g$
728.5	$\frac{1}{p}\exp(\rho p^2 + \lambda p) - \frac{1}{p}$ $\boxed{\text{S}}$ $\lambda = 0$: pair 727 $\boxed{\text{R}}$ $\rho = 0$: pair 603 $\boxed{\text{D}}$ $R(\lambda) \neq \lambda$, $R(\rho) = 0$	$\frac12\operatorname{erf}\left(\frac{g+\lambda}{2\rho^{\frac12}}\right) \mp \frac12, \qquad 0 < \pm g$
729	$\exp(\rho p^2 + \lambda p)$ $\boxed{\text{A}}$ pair 729.1 $\boxed{\text{S}}$ $\lambda = 0$: pair 710.0 $\boxed{\text{D}}$ $R(\lambda) \neq \lambda$, $R(\rho) = 0$ $\boxed{\text{D}}$ $\rho = 0$	$\frac{1}{2\pi^{\frac12}\rho^{\frac12}}\exp\left[-\frac{(g+\lambda)^2}{4\rho}\right]$
729.1	$\exp[\rho(p+\lambda)^2]$ $\boxed{\text{A}}$ pair 729 $\boxed{\text{S}}$ $\lambda = 0$: pair 710.0 $\boxed{\text{D}}$ $R(\lambda) \neq 0$, $R(\rho) = 0$ $\boxed{\text{D}}$ $\rho = 0$	$\frac{1}{2\pi^{\frac12}\rho^{\frac12}}\exp\left(-\lambda g - \frac{g^2}{4\rho}\right)$
730.1	$p^{\alpha-3}\exp(\rho p^2) - p^{\alpha-3}$ $\boxed{\text{S}}$ $\alpha = 1, 2$: pairs 727.2, 727 $\boxed{\text{B}}$ $\rho = \infty$: pair 521 $\boxed{\text{R}}$ $\rho = 0$: pair 521 $\boxed{\text{D}}$ $3 \leqq R(\alpha)$	$\frac{1}{2^{\frac12\alpha-\frac12}\pi^{\frac12}\rho^{\frac12\alpha-1}}\exp\left(-\frac{g^2}{8\rho}\right)D_{\alpha-3}\left(-\frac{g}{2^{\frac12}\rho^{\frac12}}\right) - \frac{(\frac12 \pm \frac12)}{\Gamma(3-\alpha)g^{\alpha-2}}, \quad 0 < \pm g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
731.1	$p^{\alpha-1}\exp(\rho p^2+\lambda p)$ $\boxed{S}$ $\alpha=1$ pair 729 $\boxed{S}$ $\lambda=0$ pair 740.2 $\boxed{E}$ $\rho=0$ pair 606.1 $\boxed{R}$ $2\leqq R(\alpha)$ $R(\rho)=0$ $\boxed{C}$ $R(\lambda)\neq\lambda$ $R(\rho)=0$	$\dfrac{1}{2^{\frac{1}{2}\alpha+\frac{1}{2}}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}\alpha}}\exp\left[-\dfrac{(g+\lambda)^2}{8\rho}\right]$ $\times D_{-\alpha}\left[-\dfrac{g+\lambda}{(2\rho)^{\frac{1}{2}}}\right]$
733.1	$p^{\alpha-2}\exp(\rho p^2+\lambda p)-p^{\alpha-2}$ $\boxed{S}$ $\alpha=1$ pair 728.5 $\boxed{E}$ $\rho=\infty$ pair 521 $\boxed{E}$ $\lambda=0$ pair 730.1 $\boxed{R}$ $1\leqq R(\alpha)$ $\rho=0$ and then $\lambda=0$ $\boxed{R}$ $2\leqq R(\alpha)$ $\boxed{C}$ $R(\lambda)\neq\lambda$ $R(\rho)=0$	$\dfrac{1}{2^{\frac{1}{2}\alpha}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}\alpha-\frac{1}{2}}}\exp\left[-\dfrac{(g+\lambda)^2}{8\rho}\right]$ $\times D_{1-\alpha}\left[-\dfrac{g+\lambda}{(2\rho)^{\frac{1}{2}}}\right]-\dfrac{(\frac{1}{2}\pm\frac{1}{2})}{\Gamma(2-\alpha)g^{\alpha-1}},$ $0<\pm g$
735.1	$p^{\alpha-2}\exp(\rho p^2+\lambda p)-p^{\alpha-2}\exp(\sigma p^2+\mu p)$ $\boxed{S}$ $\rho=\lambda=0$ or $\sigma=\mu=0$ pair 733.1 $\boxed{E}$ $\rho=\infty$ or $\sigma=\infty$ pair 731.1 $\boxed{E}$ $\rho=\sigma$ pair 736.1 $\boxed{E}$ $\lambda=\mu$ pair 737.1 $\boxed{R}$ $2\leqq R(\alpha)$ $\rho=0$ or $\sigma=0$ $\boxed{R}$ $3\leqq R(\alpha)$ $R(\rho)=0$ or $R(\sigma)=0$ $\boxed{C}$ $R(\lambda)\neq\lambda$ $R(\rho)=0$ $\boxed{C}$ $R(\mu)\neq\mu$ $R(\sigma)=0$	$\dfrac{1}{2^{\frac{1}{2}\alpha}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}\alpha-\frac{1}{2}}}\exp\left[-\dfrac{(g+\lambda)^2}{8\rho}\right]$ $\times D_{1-\alpha}\left[-\dfrac{g+\lambda}{(2\rho)^{\frac{1}{2}}}\right]-\dfrac{1}{2^{\frac{1}{2}\alpha}\pi^{\frac{1}{2}}\sigma^{\frac{1}{2}\alpha-\frac{1}{2}}}$ $\times\exp\left[-\dfrac{(g+\mu)^2}{8\sigma}\right]D_{1-\alpha}\left[-\dfrac{g+\mu}{(2\sigma)^{\frac{1}{2}}}\right]$
736.1	$p^{\alpha-2}(e^{\lambda p}-e^{\mu p})\exp(\rho p^2)$ $\boxed{E}$ $\lambda=\mu$ pair 731.1 $\boxed{R}$ $2\leqq R(\alpha)$ $\rho=0$ $\boxed{R}$ $3\leqq R(\alpha)$ $R(\rho)=0$ $\boxed{C}$ $R(\lambda)\neq\lambda$ and $R(\mu)\neq\mu$ $R(\rho)=0$	$\dfrac{1}{2^{\frac{1}{2}\alpha}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}\alpha-\frac{1}{2}}}\left\{\exp\left[-\dfrac{(g+\lambda)^2}{8\rho}\right]\right.$ $\times D_{1-\alpha}\left[-\dfrac{g+\lambda}{(2\rho)^{\frac{1}{2}}}\right]$ $\left.-\exp\left[-\dfrac{(g+\mu)^2}{8\rho}\right]D_{1-\alpha}\left[-\dfrac{g+\mu}{(2\rho)^{\frac{1}{2}}}\right]\right\}$

No.	Coefficient $F(f)$	Coefficient $G(g)$
737.1	$p^{\alpha-3}e^{\lambda p}[\exp(\rho p^2) - \exp(\sigma p^2)]$ [S] $\lambda = 0$; $\alpha = 1, 2, 3$: pairs 746.1, 747.1, 727.9 [B] $\rho = \infty$ or $\sigma = \infty$: pair 731.1 [R] $\rho = \sigma$: pair 731.1 [D] $3 \leqq R(\alpha)$, $\rho = 0$ or $\sigma = 0$ [D] $4 \leqq R(\alpha)$, $R(\rho) = 0$ or $R(\sigma) = 0$ [D] $R(\lambda) \neq \lambda$, $R(\rho) = 0$ or $R(\sigma) = 0$	$\frac{1}{2^{\frac{1}{2}\alpha-\frac{1}{2}}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}\alpha-1}}\exp\left[-\frac{(g+\lambda)^2}{8\rho}\right] \times D_{\alpha-3}\left[-\frac{g+\lambda}{(2\rho)^{\frac{1}{2}}}\right] - \frac{1}{2^{\frac{1}{2}\alpha-\frac{1}{2}}\pi^{\frac{1}{2}}\sigma^{\frac{1}{2}\alpha-1}}\exp\left[-\frac{(g+\lambda)^2}{8\sigma}\right] \times D_{\alpha-3}\left[-\frac{g+\lambda}{(2\sigma)^{\frac{1}{2}}}\right]$
740.1	$p^{\alpha-1}\exp(-\pi\beta f^2)$ [A] pair 740.2 [S] $\alpha = n + 1$: pair 707.0 [S] $\alpha = 1, 2, \cdots, 5$: pairs 708.0, 708.1, $\cdots$, 708.4 [B] $\beta = 0$: pair 521 [C] $R(\alpha) < 2$; $R(\beta) = 0$	$\frac{(2\pi)^{\frac{1}{2}\alpha-\frac{1}{2}}}{\beta^{\frac{1}{2}\alpha}}\exp\left(-\frac{\pi g^2}{2\beta}\right)D_{\alpha-1}\left(-\frac{2^{\frac{1}{2}}\pi^{\frac{1}{2}}g}{\beta^{\frac{1}{2}}}\right)$
740.2	$p^{\alpha-1}\exp(\rho p^2)$ [A] pair 740.1 [S] $\alpha = n + 1$: pair 709.0 [S] $\alpha = 1, 2, \cdots, 5$: pairs 710.0, 710.1, $\cdots$, 710.4 [R] $\rho = 0$: pair 521 [D] $2 \leqq R(\alpha)$, $R(\rho) = 0$	$\frac{1}{2^{\frac{1}{2}\alpha+\frac{1}{2}}\pi^{\frac{1}{2}}\rho^{\frac{1}{2}\alpha}}\exp\left(-\frac{g^2}{8\rho}\right)D_{\alpha-1}\left(-\frac{g}{2^{\frac{1}{2}}\rho^{\frac{1}{2}}}\right)$
743.1	$p^{\alpha-5}[\exp(\rho p^2) - 1 - \rho p^2]$ [S] $\alpha = 1, 2$: pairs 745.1, 744.1 [B] $\rho = \infty$: pair 521 [R] $\rho = 0$: pair 521 [D] $3 \leqq R(\alpha)$	$\frac{1}{(2\pi)^{\frac{1}{2}}(2\rho)^{\frac{1}{2}\alpha-2}}\exp\left(-\frac{g^2}{8\rho}\right)D_{\alpha-5}\left(-\frac{g}{2^{\frac{1}{2}}\rho^{\frac{1}{2}}}\right) - (\frac{1}{2} \pm \frac{1}{2})\left[\frac{1}{\Gamma(5-\alpha)g^{\alpha-4}} + \frac{\rho}{\Gamma(3-\alpha)g^{\alpha-2}}\right]$, $0 < \pm g$
744.1	$\frac{\exp(\rho p^2) - 1 - \rho p^2}{p^3}$ [D] $\rho = 0$	$\frac{\rho^{\frac{1}{2}}g}{2\pi^{\frac{1}{2}}}\exp\left(-\frac{g^2}{4\rho}\right) \mp \frac{1}{2}(\rho + \frac{1}{2}g^2)\,\mathrm{erfc}\left(\frac{\lvert g\rvert}{2\rho^{\frac{1}{2}}}\right)$, $0 < \pm g$

No	Coefficient $F(f)$	Coefficient $G(g)$
745 1	$\dfrac{\exp(\rho p^2) - 1 - \rho p^2}{p^4}$ ☒ $\rho = 0$	$\dfrac{2\rho^{\frac12}}{3\pi^{\frac12}}(\rho + \frac14 g^2)\exp\left(-\dfrac{g^2}{4\rho}\right)$ $- \frac12 \lvert g\rvert(\rho + \frac16 g^2)\operatorname{erfc}\left(\dfrac{\lvert g\rvert}{2\rho^{\frac12}}\right)$
746 1	$\dfrac{\exp(\rho p^2) - \exp(\sigma p^2)}{p^2}$ ☒ $\rho = \sigma$ pair 710 0 ☒ $\rho = -\sigma$ pair 756 ☒ $\rho = 0$ or $\sigma = 0$ pair 727 2	$\frac12 g\operatorname{erf}\left(\dfrac{g}{2\rho^{\frac12}}\right) - \frac12 g\operatorname{erf}\left(\dfrac{g}{2\sigma^{\frac12}}\right)$ $+ \dfrac{\rho^{\frac12}}{\pi^{\frac12}}\exp\left(-\dfrac{g^2}{4\rho}\right) - \dfrac{\sigma^{\frac12}}{\pi^{\frac12}}\exp\left(-\dfrac{g^2}{4\sigma}\right)$
747 1	$\dfrac{\exp(\rho p^2) - \exp(\sigma p^2)}{p}$ ☒ $\rho = -\sigma$ pair 753 ☒ $\rho = 0$ or $\sigma = 0$ pair 727 ☒ $\rho = \sigma$ pair 710 1	$\frac12\operatorname{erf}\left(\dfrac{g}{2\rho^{\frac12}}\right) - \frac12\operatorname{erf}\left(\dfrac{g}{2\sigma^{\frac12}}\right)$
751	$\sin(ap^2)$	$\dfrac{1}{2\pi^{\frac12}a^{\frac12}}\sin\left(\dfrac{g^2}{4a} - \dfrac{\pi}{4}\right)$
752	$\cos(ap^2)$	$\dfrac{1}{2\pi^{\frac12}a^{\frac12}}\cos\left(\dfrac{g^2}{4a} - \dfrac{\pi}{4}\right)$
753	$\dfrac{\sin(ap^2)}{p}$	$\frac12 S\left(\dfrac{g}{2^{\frac12}\pi^{\frac12}a^{\frac12}}\right) - \frac12 C\left(\dfrac{g}{2^{\frac12}\pi^{\frac12}a^{\frac12}}\right)$
754	$\dfrac{\cos(ap^2)}{p} - \dfrac{1}{p}$	$\frac12\left[S\left(\dfrac{g}{2^{\frac12}\pi^{\frac12}a^{\frac12}}\right) + C\left(\dfrac{g}{2^{\frac12}\pi^{\frac12}a^{\frac12}}\right) \mp 1\right]$, $0 < \pm g$
755 1	$\dfrac{\cos(ap^2)}{p + \lambda} - \dfrac{\cos(a\lambda^2)}{p + \lambda}$ ☒ $\lambda = 0$ pair 754 ☒ $a = \infty$ pairs 438 and 439	$\frac14 e^{-\lambda g}\left[\exp(ia\lambda^2)\operatorname{erf}\left(\dfrac{g}{2i^{\frac12}a^{\frac12}} - i^{\frac12}a^{\frac12}\lambda\right)\right.$ $+ \exp(-ia\lambda^2)\operatorname{erf}\left(\dfrac{g}{2i^{-\frac12}a^{\frac12}} - i^{-\frac12}a^{\frac12}\lambda\right)$ $\left.\mp 2\cos(a\lambda^2)\right]$, $0 < \pm g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
756	$\dfrac{\sin(ap^2)}{p^2}$	$\dfrac{g}{2}\left[S\left(\dfrac{g}{2^{1/2}\pi^{1/2}a^{1/2}}\right)-C\left(\dfrac{g}{2^{1/2}\pi^{1/2}a^{1/2}}\right)\right] + \dfrac{a^{1/2}}{\pi^{1/2}}\sin\left(\dfrac{g^2}{4a}+\dfrac{\pi}{4}\right)$
757	$\dfrac{\sin(ap^2)}{p^3}-\dfrac{a}{p}$	$\frac{1}{2}\left[\left(a+\dfrac{g^2}{2}\right)S\left(\dfrac{g}{2^{1/2}\pi^{1/2}a^{1/2}}\right) + \left(a-\dfrac{g^2}{2}\right)C\left(\dfrac{g}{2^{1/2}\pi^{1/2}a^{1/2}}\right) + \dfrac{a^{1/2}g}{\pi^{1/2}}\sin\left(\dfrac{g^2}{4a}+\dfrac{\pi}{4}\right)\mp a\right],$ $0 < \pm g$
758	$\sin(\pm ap^2+\lambda)$ $\boxed{\text{A}}$ pair 759 $\boxed{\text{S}}$ $\lambda = 0$: pair 751 $\boxed{\text{S}}$ $a = 1/(4\pi)$, $\lambda = \pi/8$: pair 762	$\pm\dfrac{1}{2\pi^{1/2}a^{1/2}}\sin\left(\dfrac{g^2}{4a}\pm\lambda-\dfrac{\pi}{4}\right)$
759	$\cos(\pm ap^2+\lambda)$ $\boxed{\text{A}}$ pair 758 $\boxed{\text{S}}$ $\lambda = 0$: pair 752 $\boxed{\text{S}}$ $a = 1/(4\pi)$, $\lambda = \pi/8$: pair 761	$\dfrac{1}{2\pi^{1/2}a^{1/2}}\cos\left(\dfrac{g^2}{4a}\pm\lambda-\dfrac{\pi}{4}\right)$
760	$\operatorname{cis}[\pm\pi(f^2-\frac{1}{8})]$	$\operatorname{cis}[\mp\pi(g^2-\frac{1}{8})]$
761	$\cos[\pi(f^2-\frac{1}{8})]$	$\cos[\pi(g^2-\frac{1}{8})]$
762	$\sin[\pi(f^2-\frac{1}{8})]$	$-\sin[\pi(g^2-\frac{1}{8})]$

Part 8. Other Elementary Transcendental Functions of f

No.	Coefficient $F(f)$	Coefficient $G(g)$
801	$\exp(-\alpha^{1/2}p^{1/2})$ $\boxed{\text{C}}$ $R(\alpha) = 0 \neq \alpha$ for transposed pair.	$\dfrac{\alpha^{1/2}}{2\pi^{1/2}g^{3/2}}\exp\left(-\dfrac{\alpha}{4g}\right),$ $0 < g$
802	$p\exp(-\alpha^{1/2}p^{1/2})$	$\left(\dfrac{\alpha}{2g}-3\right)\dfrac{\alpha^{1/2}}{4\pi^{1/2}g^{5/2}}\exp\left(-\dfrac{\alpha}{4g}\right).$ $0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
803	$\frac{\exp(-\sigma^{1/2}p^{1/2})}{p}-\frac{1}{p}$ ⊠ $\sigma = 0$ pair 522	$-\operatorname{erf}\left(\frac{\sigma^{1/2}}{2g^{1/2}}\right) \qquad 0 < g$
804.1	$\frac{\exp(-\sigma^{1/2}p^{1/2})}{p^2}-\frac{1}{p^2}$ $= \lim_{\gamma\to 0}\left[\frac{\exp(-\sigma^{1/2}p^{1/2})}{p(p+\gamma)}-\frac{1}{p(p+\gamma)}\right]$	$\frac{1}{2}\sigma-(g+\frac{1}{2}\sigma)\operatorname{erf}\left(\frac{\sigma^{1/2}}{2g^{1/2}}\right)$ $-\frac{\sigma^{1/2}g^{1/2}}{\pi^{1/2}}\exp\left(-\frac{\sigma}{4g}\right) \qquad 0 < g$
805	$\frac{\exp(-\sigma^{1/2}p^{1/2})}{p(p+\gamma)}-\frac{1}{p(p+\gamma)}$ ⊠ $\sigma = 0$ pair 542 ⊠ $\gamma = \infty$ pair 803 ⊠ $\gamma \to 0$ pair 804.1	$\frac{e^{-\gamma g}}{2\gamma}\left[2-\exp(i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\frac{\sigma^{1/2}}{2g^{1/2}}+i\gamma^{1/2}g^{1/2}\right)\right.$ $\left.-\exp(-i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\frac{\sigma^{1/2}}{2g^{1/2}}-i\gamma^{1/2}g^{1/2}\right)\right]$ $-\frac{1}{\gamma}\operatorname{erf}\left(\frac{\sigma^{1/2}}{2g^{1/2}}\right) \qquad 0 < g$
805.3	$\frac{\exp(-\sigma^{1/2}p^{1/2})}{p+\gamma}$ ⊠ $\sigma = 0$ pair 438 ⊠ $\gamma = \infty$ pair 801	$\frac{e^{-\gamma g}}{2}\left[\exp(-i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\frac{\sigma^{1/2}}{2g^{1/2}}-i\gamma^{1/2}g^{1/2}\right)\right.$ $\left.+\exp(i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\frac{\sigma^{1/2}}{2g^{1/2}}+i\gamma^{1/2}g^{1/2}\right)\right]$ $0 < g$
805.5	$\frac{p\exp(-\alpha^{1/2}p^{1/2})}{p+\gamma}$ ⊠ $\gamma = \infty$ pair 802 ⊠ $\gamma = 0$ pair 801 ⊠ $R(\alpha) = 0 \neq \alpha$ for transposed pair	$-\frac{\gamma e^{-\gamma g}}{2}\left[\exp(i\alpha^{1/2}\gamma^{1/2})\operatorname{erfc}\left(\frac{\alpha^{1/2}}{2g^{1/2}}+i\gamma^{1/2}g^{1/2}\right)\right.$ $\left.+\exp(-i\alpha^{1/2}\gamma^{1/2})\operatorname{erfc}\left(\frac{\alpha^{1/2}}{2g^{1/2}}-i\gamma^{1/2}g^{1/2}\right)\right]$ $+\frac{\alpha^{1/2}}{2\pi^{1/2}g^{3/2}}\exp\left(-\frac{\alpha}{4g}\right) \qquad 0 < g$
805.7	$\frac{p^{1/2}\exp(-\sigma^{1/2}p^{1/2})}{p+\gamma}$ ⊠ $\sigma = 0$ pair 541 ⊠ $\gamma = \infty$ pair 806 ⊠ $\gamma = 0$ pair 807	$\frac{\gamma^{1/2}e^{-\gamma g}}{2i}\left[\exp(i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\frac{\sigma^{1/2}}{2g^{1/2}}+i\gamma^{1/2}g^{1/2}\right)\right.$ $\left.-\exp(-i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\frac{\sigma^{1/2}}{2g^{1/2}}-i\gamma^{1/2}g^{1/2}\right)\right]$ $+\frac{1}{(\pi g)^{1/2}}\exp\left(-\frac{\sigma}{4g}\right) \qquad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
806	$p^{\frac{1}{2}}\exp(-\alpha^{\frac{1}{2}}p^{\frac{1}{2}})$ [L] $\alpha \to 0$: pair 503.1	$\left(\frac{\alpha}{2g}-1\right)\frac{1}{2\pi^{\frac{1}{2}}g^{\frac{3}{2}}}\exp\left(-\frac{\alpha}{4g}\right), \quad 0<g$
807	$\frac{\exp(-\sigma^{\frac{1}{2}}p^{\frac{1}{2}})}{p^{\frac{1}{2}}}$ [S] $\sigma = 0$: pair 522	$\frac{1}{(\pi g)^{\frac{1}{2}}}\exp\left(-\frac{\sigma}{4g}\right), \quad 0<g$
808.1	$\frac{\exp(-\sigma^{\frac{1}{2}}p^{\frac{1}{2}})}{p^{\frac{3}{2}}} = \lim_{\gamma\to 0}\left[\frac{\exp(-\sigma^{\frac{1}{2}}p^{\frac{1}{2}})}{p^{\frac{1}{2}}(p+\gamma)}\right]$	$\frac{2g^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\exp\left(-\frac{\sigma}{4g}\right)-\sigma^{\frac{1}{2}}\operatorname{erfc}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right), \quad 0<g$
809	$\frac{\exp(-\sigma^{\frac{1}{2}}p^{\frac{1}{2}})}{1+\beta^{\frac{1}{2}}p^{\frac{1}{2}}}$ [S] $\sigma = 0$: pair 543 [B] $\beta = \infty$: pair 807 [B] $\beta = 0$: pair 801	$\frac{1}{\pi^{\frac{1}{2}}\beta^{\frac{1}{2}}g^{\frac{1}{2}}}\exp\left(-\frac{\sigma}{4g}\right)$ $-\frac{1}{\beta^{3}}\exp\left(\frac{\sigma^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}+\frac{g}{\beta^{3}}\right)\operatorname{erfc}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\frac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right),$ $0<g$
810	$\frac{p\exp(-\alpha^{\frac{1}{2}}p^{\frac{1}{2}})}{1+\beta^{\frac{1}{2}}p^{\frac{1}{2}}}$ [B] $\beta = \infty$: pair 806 [B] $\beta = 0$: pair 802	$\left(\frac{\alpha}{4g^{2}}-\frac{\alpha^{\frac{1}{2}}+\beta^{\frac{1}{2}}}{2\beta^{\frac{1}{2}}g}+\frac{1}{\beta^{3}}\right)\frac{1}{\pi^{\frac{1}{2}}\beta^{\frac{1}{2}}g^{\frac{1}{2}}}\exp\left(-\frac{\alpha}{4g}\right)$ $-\frac{1}{\beta^{6}}\exp\left(\frac{\alpha^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}+\frac{g}{\beta^{3}}\right)$ $\times\operatorname{erfc}\left(\frac{\alpha^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\frac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right), \quad 0<g$
811	$\frac{\exp(-\sigma^{\frac{1}{2}}p^{\frac{1}{2}})}{p(1+\beta^{\frac{1}{2}}p^{\frac{1}{2}})}-\frac{1}{p}$ [S] $\sigma = 0$: pair 551 [B] $\beta = 0$: pair 803.	$-\exp\left(\frac{\sigma^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}+\frac{g}{\beta^{3}}\right)\operatorname{erfc}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\frac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right)$ $-\operatorname{erf}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right), \quad 0<g$
812	$\frac{\exp(-\sigma^{\frac{1}{2}}p^{\frac{1}{2}})}{(p+\gamma)(1+\beta^{\frac{1}{2}}p^{\frac{1}{2}})}$ [S] $\sigma = 0$: pair 545.5 [B] $\beta = \infty$: pair 812.5 [B] $\gamma = \infty$: pair 809 [B] $\beta = 0$: pair 805.3	$\frac{e^{-\gamma g}}{2}\left[\frac{\exp(i\gamma^{\frac{1}{2}}\sigma^{\frac{1}{2}})}{1-i\beta^{\frac{1}{2}}\gamma^{\frac{1}{2}}}\operatorname{erfc}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+i\gamma^{\frac{1}{2}}g^{\frac{1}{2}}\right)\right.$ $\left.+\frac{\exp(-i\gamma^{\frac{1}{2}}\sigma^{\frac{1}{2}})}{1+i\beta^{\frac{1}{2}}\gamma^{\frac{1}{2}}}\operatorname{erfc}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}-i\gamma^{\frac{1}{2}}g^{\frac{1}{2}}\right)\right]$ $-\frac{1}{1+\beta^{3}\gamma}\exp\left(\frac{\sigma^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}+\frac{g}{\beta^{3}}\right)$ $\times\operatorname{erfc}\left(\frac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\frac{g^{\frac{1}{2}}}{\beta^{\frac{1}{2}}}\right), \quad 0<g$

No	Coefficient $F(f)$	Coefficient $G(g)$
812.5	$\dfrac{\exp(-\sigma^{1/2}p^{1/2})}{p^{1/2}(p+\gamma)}$ S $\sigma = 0$ pair 542 B $\gamma = \infty$ pair 807 L $\gamma \to 0$ pair 808.1	$\dfrac{e^{-\gamma g}}{2\gamma^{1/2}}\Big[\exp(i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} + i\gamma^{1/2}g^{1/2}\Big)$ $- \exp(-i\gamma^{1/2}\sigma^{1/2})\operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} - i\gamma^{1/2}g^{1/2}\Big)\Big]$ $0 < g$
813	$\dfrac{p\exp(-\sigma^{1/2}p^{1/2})}{(p+\gamma)(1+\beta p^{1/2})}$ S $\sigma = 0$ pair 545 B $\beta = \infty$ pair 805.7 B $\gamma = \infty$ pair 810 B $\beta = 0$ pair 805.5 B $\gamma = 0$ pair 809	$-\dfrac{\gamma e^{-\gamma g}}{2}\Big[\dfrac{\exp(i\gamma^{1/2}\sigma^{1/2})}{1 - i\beta\gamma^{1/2}}\operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} + i\gamma^{1/2}g^{1/2}\Big)$ $+ \dfrac{\exp(-i\gamma^{1/2}\sigma^{1/2})}{1 + i\beta\gamma^{1/2}}\operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} - i\gamma^{1/2}g^{1/2}\Big)\Big]$ $- \dfrac{1}{\beta(1+\beta^2\gamma)}\exp\Big(\dfrac{\sigma^{1/2}}{\beta} + \dfrac{g}{\beta^2}\Big)$ $\times \operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} + \dfrac{g^{1/2}}{\beta}\Big)$ $+ \dfrac{1}{\pi^{1/2}\beta g^{1/2}}\exp\Big(-\dfrac{\sigma}{4g}\Big) \quad 0 < g$
814	$\dfrac{p^{1/2}\exp(-\alpha^{1/2}p^{1/2})}{1+\beta p^{1/2}}$ B $\beta = \infty$ pair 801 B $\beta = 0$ pair 806 C $R(\alpha) = 0 \ddagger \alpha$ for transposed pair	$\Big(\dfrac{\alpha^{1/2}}{2g} - \dfrac{1}{\beta}\Big)\dfrac{1}{\pi^{1/2}\beta g^{1/2}}\exp\Big(-\dfrac{\alpha}{4g}\Big)$ $+ \dfrac{1}{\beta^2}\exp\Big(\dfrac{\alpha^{1/2}}{\beta} + \dfrac{g}{\beta^2}\Big)\operatorname{erfc}\Big(\dfrac{\alpha^{1/2}}{2g^{1/2}} + \dfrac{g^{1/2}}{\beta}\Big)$ $0 < g$
815	$\dfrac{\exp(-\sigma^{1/2}p^{1/2})}{p^{1/2}(1+\beta p^{1/2})}$ S $\sigma = 0$ pair 551 B $\beta = 0$ pair 807	$\dfrac{1}{\beta}\exp\Big(\dfrac{\sigma^{1/2}}{\beta} + \dfrac{g}{\beta^2}\Big)\operatorname{erfc}\Big(\dfrac{g^{1/2}}{\beta} + \dfrac{\sigma^{1/2}}{2g^{1/2}}\Big)$ $0 < g$
815.5	$\dfrac{\exp(-\sigma^{1/2}p^{1/2})}{p^{1/2}(p+\gamma)(1+\beta p^{1/2})}$ S $\sigma = 0$ pair 5[illegible]2.3 B $\gamma = \infty$ pair 8[illegible]5 B $\beta = 0$ pair 812.5	$\dfrac{e^{-\gamma g}}{2\gamma^{1/2}}\Big[\dfrac{\exp(i\gamma^{1/2}\sigma^{1/2})}{1 - i\beta\gamma^{1/2}}\operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} + i\gamma^{1/2}g^{1/2}\Big)$ $- \dfrac{\exp(-i\gamma^{1/2}\sigma^{1/2})}{1 + i\beta\gamma^{1/2}}\operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} - i\gamma^{1/2}g^{1/2}\Big)\Big]$ $+ \dfrac{\beta^2}{1+\beta^2\gamma}\exp\Big(\dfrac{\sigma^{1/2}}{\beta} + \dfrac{g}{\beta^2}\Big)$ $\times \operatorname{erfc}\Big(\dfrac{\sigma^{1/2}}{2g^{1/2}} + \dfrac{g^{1/2}}{\beta}\Big) \quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
816	$\dfrac{p^{1/2}\exp(-\sigma^{1/2}p^{1/2})}{(p+\gamma)(1+\beta^{1/2}p^{1/2})}$ [S] $\sigma = 0$: pair 552 [B] $\beta = \infty$: pair 805.3 [B] $\gamma = \infty$: pair 814 [B] $\beta = 0$: pair 805.7 [B] $\gamma = 0$: pair 815	$\dfrac{\gamma^{1/2}e^{-\gamma g}}{2i}\left[\dfrac{\exp(i\gamma^{1/2}\sigma^{1/2})}{1-i\beta^{1/2}\gamma^{1/2}}\operatorname{erfc}\left(\dfrac{\sigma^{1/2}}{2g^{1/2}}+i\gamma^{1/2}g^{1/2}\right)\right.$ $\left.-\dfrac{\exp(-i\gamma^{1/2}\sigma^{1/2})}{1+i\beta^{1/2}\gamma^{1/2}}\operatorname{erfc}\left(\dfrac{\sigma^{1/2}}{2g^{1/2}}-i\gamma^{1/2}g^{1/2}\right)\right]$ $+\dfrac{1}{\beta^{1/2}(1+\beta\gamma)}\exp\left(\dfrac{\sigma^{1/2}}{\beta^{1/2}}+\dfrac{g}{\beta}\right)$ $\times\operatorname{erfc}\left(\dfrac{\sigma^{1/2}}{2g^{1/2}}+\dfrac{g^{1/2}}{\beta^{1/2}}\right),\quad 0<g$
816.5	$(p+\rho)\exp[-\alpha^{1/2}(p+\rho)^{1/2}]$ [S] $\rho = 0$: pair 802	$\left(\dfrac{\alpha}{2g}-3\right)\dfrac{\alpha^{1/2}}{4\pi^{1/2}g^{5/2}}\exp\left(-\rho g-\dfrac{\alpha}{4g}\right),$ $0<g$
817	$\exp[-\alpha^{1/2}(p+\rho)^{1/2}]$ [S] $\rho = 0$: pair 801 [C] $R(\alpha) = 0 \neq \alpha$ for transposed pair.	$\dfrac{\alpha^{1/2}}{2\pi^{1/2}g^{3/2}}\exp\left(-\rho g-\dfrac{\alpha}{4g}\right),\quad 0<g$
818.1	$\dfrac{\exp[-\sigma^{1/2}(p+\rho)^{1/2}]}{p}-\dfrac{\exp(-\rho^{1/2}\sigma^{1/2})}{p}$ [S] $\rho = 0$: pair 803 [S] $\sigma = 0$: pair 548.1	$\tfrac{1}{2}\left[\exp(\rho^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\rho^{1/2}g^{1/2}+\dfrac{\sigma^{1/2}}{2g^{1/2}}\right)\right.$ $\left.-\exp(-\rho^{1/2}\sigma^{1/2})\operatorname{erfc}\left(\rho^{1/2}g^{1/2}-\dfrac{\sigma^{1/2}}{2g^{1/2}}\right)\right],$ $0<g$
819	$\dfrac{\exp[-\sigma^{1/2}(p+\rho)^{1/2}]}{p+\gamma}$ [S] $\rho = 0$: pair 805.3 [S] $\sigma = 0$: pair 438 [S] $\rho = \gamma$: pair 824.7 [B] $\gamma = \infty$: pair 817 [B] $\rho = \infty$: pair 438	$\dfrac{e^{-\gamma g}}{2}\left\{\exp[-\sigma^{1/2}(\rho-\gamma)^{1/2}]\right.$ $\times\operatorname{erfc}\left[\dfrac{\sigma^{1/2}}{2g^{1/2}}-(\rho-\gamma)^{1/2}g^{1/2}\right]$ $+\exp[\sigma^{1/2}(\rho-\gamma)^{1/2}]$ $\left.\times\operatorname{erfc}\left[\dfrac{\sigma^{1/2}}{2g^{1/2}}+(\rho-\gamma)^{1/2}g^{1/2}\right]\right\},\quad 0<g$
820	$(p+\rho)^{1/2}\exp[-\alpha^{1/2}(p+\rho)^{1/2}]$ [S] $\rho = 0$: pair 806 [L] $\alpha \to 0$: pair 506	$\left(\dfrac{\alpha}{2g}-1\right)\dfrac{1}{2\pi^{1/2}g^{3/2}}\exp\left(-\rho g-\dfrac{\alpha}{4g}\right),$ $0<g$

No	Coefficient $F(f)$	Coefficient $G(g)$
821.1	$\dfrac{(p+\rho)^{1/2}\exp[-\sigma^{1/2}(p+\rho)^{1/2}]}{p} - \dfrac{\rho^{1/2}\exp(-\rho^{1/2}\sigma^{1/2})}{p}$ $\boxed{S}$ $\rho = 0$ pair 807 $\boxed{S}$ $\sigma = 0$ pair 548.1	$\dfrac{1}{(\pi g)^{1/2}}\exp\left(-\rho g - \dfrac{\sigma}{4g}\right) - \dfrac{\rho^{1/2}}{2}\left[\exp(\rho^{1/2}\sigma^{1/2})\,\mathrm{erfc}\left(\rho^{1/2}g^{1/2} + \dfrac{\sigma^{1/2}}{2g^{1/2}}\right) + \exp(-\rho^{1/2}\sigma^{1/2})\,\mathrm{erfc}\left(\rho^{1/2}g^{1/2} - \dfrac{\sigma^{1/2}}{2g^{1/2}}\right)\right]$ $0 < g$
822	$\dfrac{(p+\rho)^{1/2}\exp[-\sigma^{1/2}(p+\rho)^{1/2}]}{p+\gamma}$ $\boxed{S}$ $\rho = 0$ pair 805.7 $\boxed{S}$ $\sigma = 0$ pair 549 $\boxed{B}$ $\gamma = \infty$ pair 820 $\boxed{B}$ $\rho = \infty$ pair 438 $\boxed{B}$ $\gamma = \rho$ pair 823	$\dfrac{(\rho-\gamma)^{1/2}e^{-\gamma g}}{2}\left\{\exp[-\sigma^{1/2}(\rho-\gamma)^{1/2}] \times \mathrm{erfc}\left[\dfrac{\sigma^{1/2}}{2g^{1/2}} - (\rho-\gamma)^{1/2}g^{1/2}\right] - \exp[\sigma^{1/2}(\rho-\gamma)^{1/2}] \times \mathrm{erfc}\left[\dfrac{\sigma^{1/2}}{2g^{1/2}} + (\rho-\gamma)^{1/2}g^{1/2}\right]\right\} + \dfrac{1}{(\pi g)^{1/2}}\exp\left(-\rho g - \dfrac{\sigma}{4g}\right)$ $\quad 0 < g$
822.5	$\dfrac{(p+\rho)\exp[-\alpha^{1/2}(p+\rho)^{1/2}]}{p+\gamma}$ $\boxed{S}$ $\rho = 0$ pair 805.5 $\boxed{B}$ $\gamma = \infty$ pair 816.5 $\boxed{B}$ $\rho = \infty$ pair 438 $\boxed{B}$ $\gamma = \rho$ pair 817 $\boxed{C}$ $R(\alpha) = 0 \neq \alpha$ for transposed pair	$\dfrac{(\rho-\gamma)e^{-\gamma g}}{2}\left\{\exp[\alpha^{1/2}(\rho-\gamma)^{1/2}] \times \mathrm{erfc}\left[\dfrac{\alpha^{1/2}}{2g^{1/2}} + (\rho-\gamma)^{1/2}g^{1/2}\right] + \exp[-\alpha^{1/2}(\rho-\gamma)^{1/2}] \times \mathrm{erfc}\left[\dfrac{\alpha^{1/2}}{2g^{1/2}} - (\rho-\gamma)^{1/2}g^{1/2}\right]\right\} + \dfrac{\alpha^{1/2}}{2\pi^{1/2}g^{3/2}}\exp\left(-\rho g - \dfrac{\alpha}{4g}\right)$, $\quad 0 < g$
823	$\dfrac{1}{(p+\rho)^{1/2}}\exp[-\sigma^{1/2}(p+\rho)^{1/2}]$ $\boxed{S}$ $\rho = 0$ pair 807 $\boxed{S}$ $\sigma = 0$ pair 526	$\dfrac{1}{(\pi g)^{1/2}}\exp\left(-\rho g - \dfrac{\sigma}{4g}\right)$, $\quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
823.3	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]-\exp[-\tau^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}}}$ [S] $\sigma = 0$ or $\tau = 0$: pair 823.5 [B] $\sigma = \infty$ or $\tau = \infty$: pair 823 [R] $\sigma = \tau$: pair 817	$\dfrac{1}{(\pi g)^{\frac{1}{2}}} e^{-\rho g}\left[\exp\left(-\dfrac{\sigma}{4g}\right) - \exp\left(-\dfrac{\tau}{4g}\right)\right], \quad 0 < g$
823.5	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]-1}{(p+\rho)^{\frac{1}{2}}}$ [B] $\sigma = \infty$: pair 526 [D] $\sigma = 0$	$\dfrac{1}{(\pi g)^{\frac{1}{2}}} e^{-\rho g}\left[\exp\left(-\dfrac{\sigma}{4g}\right) - 1\right], \quad 0 < g$
824.1	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{p(p+\rho)^{\frac{1}{2}}} - \dfrac{\exp(-\rho^{\frac{1}{2}}\sigma^{\frac{1}{2}})}{\rho^{\frac{1}{2}}p}$ [S] $\sigma = 0$: pair 547.1 [D] $\rho = 0$	$-\dfrac{1}{2\rho^{\frac{1}{2}}}\left[\exp(\rho^{\frac{1}{2}}\sigma^{\frac{1}{2}})\operatorname{erfc}\left(\rho^{\frac{1}{2}}g^{\frac{1}{2}} + \dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right) + \exp(-\rho^{\frac{1}{2}}\sigma^{\frac{1}{2}})\operatorname{erfc}\left(\rho^{\frac{1}{2}}g^{\frac{1}{2}} - \dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right)\right],$ $0 < g$
824.5	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\beta)^{\frac{1}{2}}]}{(p+\beta)^{\frac{1}{2}}}$ [S] $\sigma = 0$: pair 529	$e^{-\beta g}\left[\dfrac{2g^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\exp\left(-\dfrac{\sigma}{4g}\right) - \sigma^{\frac{1}{2}}\operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right)\right], \quad 0 < g$
824.7	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\beta)^{\frac{1}{2}}]}{p+\beta}$ [S] $\sigma = 0$: pair 438	$e^{-\beta g}\operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right), \quad 0 < g$
824.9	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\beta)^{\frac{1}{2}}]}{(p+\beta)^2}$ [S] $\sigma = 0$: pair 442	$e^{-\beta g}\left[(g + \tfrac{1}{2}\sigma)\operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right) - \dfrac{\sigma^{\frac{1}{2}}g^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\exp\left(-\dfrac{\sigma}{4g}\right)\right] \quad 0 < g$
825	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\gamma)(p+\rho)^{\frac{1}{2}}}$ [S] $\rho = 0$: pair 812.5 [S] $\sigma = 0$: pair 546 [S] $\rho = \gamma$: pair 824.5 [B] $\gamma = \infty$: pair 823 [B] $\rho = \infty$: pair 438	$\dfrac{e^{-\gamma g}}{2(\rho-\gamma)^{\frac{1}{2}}}\left\{\exp[-\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}] \times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}} - (\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right] - \exp[\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}] \times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}} + (\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]\right\}, \quad 0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
825.5	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\beta)^{\frac{1}{2}}]}{(p+\beta)(p+\gamma)}$ $\sigma = 0$ pair 448 $\beta = \gamma$ pair 824.9 $\beta = \infty$ pair 438 $\gamma = \infty$ pair 824.7	$\dfrac{e^{-\gamma g}}{2(\beta-\gamma)}\Big\{\exp[\sigma^{\frac{1}{2}}(\beta-\gamma)^{\frac{1}{2}}]$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+(\beta-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $+\exp[-\sigma^{\frac{1}{2}}(\beta-\gamma)^{\frac{1}{2}}]$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}-(\beta-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]\Big\}$ $-\dfrac{e^{-\beta g}}{\beta-\gamma}\operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right) \quad 0 < g$
826.1	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}}(p+\gamma)[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\lambda = 0$ pair 825.5 $\rho = 0$ pair 815.5 $\sigma = 0$ pair 552.5 $\rho = \gamma$ pair 834.1 $\gamma = \infty$ pair 830.0 $\lambda = \infty$ pair 825 $\rho = \infty$ pair 438 $R(\lambda) \leqq -\{R(\rho)+[I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}-\gamma g]}{2(\rho-\gamma)^{\frac{1}{2}}[\lambda+(\rho-\gamma)^{\frac{1}{2}}]}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}-(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $-\dfrac{\exp[\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}-\gamma g]}{2(\rho-\gamma)^{\frac{1}{2}}[\lambda-(\rho-\gamma)^{\frac{1}{2}}]}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $+\dfrac{\exp[\lambda\sigma^{\frac{1}{2}}+\lambda^2 g-\rho g]}{\lambda^2+\gamma-\rho}$ $\times \operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\lambda g^{\frac{1}{2}}\right) \quad 0 < g$
827.1	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\gamma)[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\rho = 0$ pair 812 $\sigma = 0$ pair 545.7 $\rho = \gamma$ pair 833.1 $\gamma = \infty$ pair 832.1 $\lambda = \infty$ pair 819 $\rho = \infty$ pair 438 $\lambda = 0$ pair 825 $R(\lambda) \leqq -\{R(\rho)+[I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}-\gamma g]}{2[\lambda+(\rho-\gamma)^{\frac{1}{2}}]}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}-(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $+\dfrac{\exp[\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}-\gamma g]}{2[\lambda-(\rho-\gamma)^{\frac{1}{2}}]}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $-\dfrac{\lambda\exp(\lambda\sigma^{\frac{1}{2}}+\lambda^2 g-\rho g)}{\lambda^2+\gamma-\rho}$ $\times \operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\lambda g^{\frac{1}{2}}\right) \quad 0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
828.1	$\dfrac{(p+\rho)^{\frac{1}{2}}\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\gamma)[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\boxed{S}$ $\rho = 0$: pair 816 $\boxed{S}$ $\sigma = 0$: pair 552.1 $\boxed{B}$ $\gamma = \infty$: pair 835.1 $\boxed{B}$ $\lambda = \infty$: pair 822 $\boxed{B}$ $\rho = \infty$: pair 438 $\boxed{B}$ $\gamma = \rho$: pair 830.0 $\boxed{R}$ $\lambda = 0$: pair 819 $\boxed{O}$ $R(\lambda) \leqq -\{R(\rho)+[I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{(\rho-\gamma)^{\frac{1}{2}}e^{-\gamma g}}{2}\Bigg\{\dfrac{\exp[-\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}]}{\lambda+(\rho-\gamma)^{\frac{1}{2}}}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}-(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $-\dfrac{\exp[\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}]}{\lambda-(\rho-\gamma)^{\frac{1}{2}}}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]\Bigg\}$ $+\dfrac{\lambda^2\exp(\lambda\sigma^{\frac{1}{2}}+\lambda^2 g-\rho g)}{\lambda^2+\gamma-\rho}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\lambda g^{\frac{1}{2}}\right], \quad 0<g$
829.1	$\dfrac{(p+\rho)\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\gamma)[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\boxed{S}$ $\rho = 0$: pair 813 $\boxed{S}$ $\sigma = 0$: pair 545.2 $\boxed{B}$ $\gamma = \infty$: pair 836.1 $\boxed{B}$ $\lambda = \infty$: pair 822.5 $\boxed{B}$ $\rho = \infty$: pair 438 $\boxed{B}$ $\gamma = \rho$: pair 832.1 $\boxed{R}$ $\lambda = 0$: pair 822 $\boxed{O}$ $R(\lambda) \leqq -\{R(\rho)+[I(\lambda)]^2\}^{\frac{1}{2}}$	$\dfrac{(\rho-\gamma)e^{-\gamma g}}{2}\Bigg\{\dfrac{\exp[\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}]}{\lambda-(\rho-\gamma)^{\frac{1}{2}}}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]$ $+\dfrac{\exp[-\sigma^{\frac{1}{2}}(\rho-\gamma)^{\frac{1}{2}}]}{\lambda+(\rho-\gamma)^{\frac{1}{2}}}$ $\times \operatorname{erfc}\left[\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}-(\rho-\gamma)^{\frac{1}{2}}g^{\frac{1}{2}}\right]\Bigg\}$ $-\dfrac{\lambda^3\exp(\lambda\sigma^{\frac{1}{2}}+\lambda^2 g-\rho g)}{\lambda^2+\gamma-\rho}$ $\times \operatorname{erfc}\left(\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}+\lambda g^{\frac{1}{2}}\right)$ $+\dfrac{1}{(\pi g)^{\frac{1}{2}}}\exp\left(-\rho g-\dfrac{\sigma}{4g}\right), \quad 0<g$
830.0 Key	$\dfrac{\exp[-\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}}[\lambda+(p+\rho)^{\frac{1}{2}}]}$ $\boxed{S}$ $\lambda = 0$: pair 824.7 $\boxed{S}$ $\rho = 0$: pair 815 $\boxed{S}$ $\sigma = 0$: pair 551.5 $\boxed{S}$ $\lambda^2 = \rho$: pair 831.1 $\boxed{B}$ $\lambda = \infty$: pair 823 $\boxed{O}$ $R(\lambda) \leqq -\{R(\rho)+[I(\lambda)]^2\}^{\frac{1}{2}}$	$\exp(\lambda\sigma^{\frac{1}{2}}+\lambda^2 g-\rho g)\operatorname{erfc}\left(\lambda g^{\frac{1}{2}}+\dfrac{\sigma^{\frac{1}{2}}}{2g^{\frac{1}{2}}}\right),$ $0<g$

No	Coefficient $F(f)$	Coefficient $G(g)$
831.1	$\left[\frac{1}{p(p+\rho)^{\frac12}} - \frac{1}{\rho^{\frac12}p}\right]\exp[-\sigma^{\frac12}(p+\rho)^{\frac12}]$ ⊠ $\sigma = 0$ pair 547.1 ⊠ $\rho = 0$	$-\frac{1}{\rho^{\frac12}}\exp(\rho^{\frac12}\sigma^{\frac12})\operatorname{erfc}\left(\rho^{\frac12}g^{\frac12} + \frac{\sigma^{\frac12}}{2g^{\frac12}}\right)$ $0 < g$
832.1	$\frac{\exp[-\sigma^{\frac12}(p+\rho)^{\frac12}]}{\lambda + (p+\rho)^{\frac12}}$ ⊠ $\rho = 0$ pair 809 ⊠ $\sigma = 0$ pair 543.5 ⊠ $\lambda = \infty$ pair 817 ⊠ $\lambda = 0$ pair 823 ⊠ $R(\lambda) \leqq -\{R(\rho) + [I(\lambda)]^2\}^{\frac12}$	$e^{-\rho g}\left[\frac{1}{(\pi g)^{\frac12}}\exp\left(-\frac{\sigma}{4g}\right)\right.$ $-\lambda\exp(\lambda\sigma^{\frac12} + \lambda^2 g)$ $\left.\times \operatorname{erfc}\left(\frac{\sigma^{\frac12}}{2g^{\frac12}} + \lambda g^{\frac12}\right)\right],\quad 0 < g$
833.1	$\frac{\exp[-\sigma^{\frac12}(p+\beta)^{\frac12}]}{(p+\beta)[\lambda + (p+\beta)^{\frac12}]}$ ⊠ $\lambda = 0$ pair 824.3 ⊠ $\sigma = 0$ pair 545.9 ⊠ $\lambda = \infty$ pair 824.7 ⊠ $R(\lambda) \leqq -\{R(\beta) + [I(\lambda)]^2\}^{\frac12}$	$\frac{e^{-\beta g}}{\lambda}\left[\operatorname{erfc}\left(\frac{\sigma^{\frac12}}{2g^{\frac12}}\right)\right.$ $-\exp(\lambda\sigma^{\frac12} + \lambda^2 g)$ $\left.\times \operatorname{erfc}\left(\frac{\sigma^{\frac12}}{2g^{\frac12}} + \lambda g^{\frac12}\right)\right]\quad 0 < g$
834.1	$\frac{\exp[-\sigma^{\frac12}(p+\beta)^{\frac12}]}{(p+\beta)^{\frac12}[\lambda + (p+\beta)^{\frac12}]}$ ⊠ $\lambda = 0$ pair 824.9 ⊠ $\sigma = 0$ pair 552.9 ⊠ $\lambda = \infty$ pair 824.5 ⊠ $R(\lambda) \leqq -\{R(\beta) + [I(\lambda)]^2\}^{\frac12}$	$\frac{e^{-\beta g}}{\lambda^2}\left[\exp(\lambda\sigma^{\frac12} + \lambda^2 g)\operatorname{erfc}\left(\frac{\sigma^{\frac12}}{2g^{\frac12}} + \lambda g^{\frac12}\right)\right.$ $-(1 + \lambda\sigma^{\frac12})\operatorname{erfc}\left(\frac{\sigma^{\frac12}}{2g^{\frac12}}\right)$ $\left.+\frac{2\lambda g^{\frac12}}{\pi^{\frac12}}\exp\left(-\frac{\sigma}{4g}\right)\right],\quad 0 < g$
835.1	$\frac{(p+\rho)^{\frac12}\exp[-\alpha^{\frac12}(p+\rho)^{\frac12}]}{\lambda + (p+\rho)^{\frac12}}$ ⊠ $\rho = 0$ pair 814 ⊠ $\lambda = \infty$ pair 820 ⊠ $\lambda = 0$ pair 817 ⊠ $R(\alpha) = 0 \neq \alpha$ for transposed pair ⊠ $R(\lambda) \leqq -\{R(\rho) + [I(\lambda)]^2\}^{\frac12}$	$\left(\frac{\alpha^{\frac12}}{2g} - \lambda\right)\frac{1}{(\pi g)^{\frac12}}\exp\left(-\rho g - \frac{\alpha}{4g}\right)$ $+\lambda^2\exp(\lambda\alpha^{\frac12} + \lambda^2 g - \rho g)$ $\times \operatorname{erfc}\left(\frac{\alpha^{\frac12}}{2g^{\frac12}} + \lambda g^{\frac12}\right)\quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
836.1	$\dfrac{(p+\rho)\exp[-\alpha^{1/2}(p+\rho)^{1/2}]}{\lambda+(p+\rho)^{1/2}}$ $\boxed{S}$ $\rho = 0$: pair 810 $\boxed{B}$ $\lambda = \infty$: pair 816.5 $\boxed{R}$ $\lambda = 0$: pair 820 $\boxed{D}$ $R(\lambda) \leqq -\{R(\rho)+[I(\lambda)]^2\}^{1/2}$	$e^{-\rho g}\Big[\Big(\dfrac{\alpha}{4g^2}-\dfrac{\lambda\alpha^{1/2}+1}{2g}+\lambda^2\Big)\dfrac{1}{(\pi g)^{1/2}}$ $\times \exp\Big(-\dfrac{\alpha}{4g}\Big)-\lambda^3\exp(\lambda\alpha^{1/2}+\lambda^2 g)$ $\times \operatorname{erfc}\Big(\dfrac{\alpha^{1/2}}{2g^{1/2}}+\lambda g^{1/2}\Big)\Big],\quad 0<g$
840.1	$\dfrac{\lvert p\rvert}{p}\exp(-\lambda\lvert p\rvert^{1/2}-\rho\lvert p\rvert)$ $\boxed{S}$ $\lambda = 0$: pair 638.1 $\boxed{S}$ $\rho = 0$: pair 840.3 $\boxed{D}$ $R(\lambda) \leqq 0,\ R(\rho) = 0$	$\dfrac{g}{\pi(\rho^2+g^2)}$ $-\dfrac{i\lambda}{4\pi^{1/2}(\rho+ig)^{3/2}}\exp\Big[\dfrac{\lambda^2}{4(\rho+ig)}\Big]$ $\times \operatorname{erfc}\Big[\dfrac{\lambda}{2(\rho+ig)^{1/2}}\Big]$ $+\dfrac{i\lambda}{4\pi^{1/2}(\rho-ig)^{3/2}}\exp\Big[\dfrac{\lambda^2}{4(\rho-ig)}\Big]$ $\times \operatorname{erfc}\Big[\dfrac{\lambda}{2(\rho-ig)^{1/2}}\Big]$
840.3	$\dfrac{\lvert p\rvert}{p}\exp(-\alpha\lvert p\rvert^{1/2})$	$\dfrac{1}{\pi g}\pm\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}\Big\{\Big[\tfrac{1}{2}-C\Big(\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\Big)\Big]$ $\times \sin\Big(\dfrac{\alpha^2}{4\lvert g\rvert}\Big)$ $-\Big[\tfrac{1}{2}-S\Big(\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\Big)\Big]\cos\Big(\dfrac{\alpha^2}{4\lvert g\rvert}\Big)\Big\},$ $0<\pm g$
841	$\dfrac{1}{\lvert p\rvert^{1/2}}\exp(-\rho\lvert p\rvert^{1/2})$ $\boxed{S}$ $\rho = 0$: pair 523.1	$\dfrac{2^{1/2}}{\pi^{1/2}\lvert g\rvert^{1/2}}\Big\{\Big[\tfrac{1}{2}-S\Big(\dfrac{\rho}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\Big)\Big]$ $\times\cos\Big(\dfrac{\rho^2}{4\lvert g\rvert}\Big)$ $-\Big[\tfrac{1}{2}-C\Big(\dfrac{\rho}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\Big)\Big]\sin\Big(\dfrac{\rho^2}{4\lvert g\rvert}\Big)\Big\}$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
842	$\frac{1}{\lvert p\rvert^{1/2}}\exp(-\lambda\lvert p\rvert^{1/2}-\rho\lvert p\rvert)$ ⊠ $\lambda = 0$ pair 634 ⊠ $\rho = 0$ pair 841 ⊠ $R(\lambda) < 0$ $R(\rho) = 0$	$\frac{1}{2\pi^{1/2}(\rho+ig)^{1/2}}\exp\left[\frac{\lambda}{4(\rho+ig)}\right]\times\operatorname{erfc}\left[\frac{\lambda}{2(\rho+ig)^{1/2}}\right]$ $+\frac{1}{2\pi^{1/2}(\rho-ig)^{1/2}}\exp\left[\frac{\lambda^2}{4(\rho-ig)}\right]\times\operatorname{erfc}\left[\frac{\lambda}{2(\rho-ig)^{1/2}}\right]$
843	$\lvert p\rvert^{1/2}\sin(\lambda\lvert p\rvert^{1/2})-\lim_{\beta\to 0}[\lvert p\rvert^{1/2}\sin(\lambda\lvert p\rvert^{1/2})e^{\beta p}]$ ⊠ $\lambda = 0$ pair 407	$-\frac{\lambda}{2\pi g}-\frac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}C\left(\frac{\lambda}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\times\left[\cos\left(\frac{\lambda}{4\lvert g\rvert}\right)-\frac{\lambda^2}{2\lvert g\rvert}\sin\left(\frac{\lambda^2}{4\lvert g\rvert}\right)\right]$ $-\frac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}S\left(\frac{\lambda}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\times\left[\sin\left(\frac{\lambda^2}{4\lvert g\rvert}\right)+\frac{\lambda^2}{2\lvert g\rvert}\cos\left(\frac{\lambda^2}{4\lvert g\rvert}\right)\right]$
844	$\frac{\lvert p\rvert^{1/2}}{p}\sin(x\lvert p\rvert^{1/2})$ ⊠ $x = 0$	$\pm\frac{2^{1/2}}{\pi^{1/2}\lvert g\rvert^{1/2}}\left[C\left(\frac{x}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\cos\left(\frac{x^2}{4\lvert g\rvert}\right)+S\left(\frac{x}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\sin\left(\frac{x^2}{4\lvert g\rvert}\right)\right]$ $0 < \pm g$
845	$\cos(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}$ ⊠ $\lambda = 0$ pair 632 ⊠ $\beta \to 0$ pair 845.2	$\frac{i\lambda}{4\pi^{1/2}(\beta+ig)^{3/2}}\exp\left[-\frac{\lambda^2}{4(\beta+ig)}\right]\times\operatorname{erf}\left[\frac{i\lambda}{2(\beta+ig)^{1/2}}\right]$ $+\frac{i\lambda}{4\pi^{1/2}(\beta-ig)^{3/2}}\exp\left[-\frac{\lambda^2}{4(\beta-ig)}\right]\times\operatorname{erf}\left[\frac{i\lambda}{2(\beta-ig)^{1/2}}\right]$ $+\frac{\beta}{\pi(\beta^2+g^2)}$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
845.2	$\cos(\lambda\|p\|^{\frac{1}{2}}) = \lim\limits_{\beta\to 0}[\cos(\lambda\|p\|^{\frac{1}{2}})e^{-\beta\|p\|}]$	$\dfrac{\lambda}{2^{\frac{1}{2}}\pi^{\frac{1}{2}}\|g\|^{\frac{3}{2}}}\left[C\left(\dfrac{\lambda}{2^{\frac{1}{2}}\pi^{\frac{1}{2}}\|g\|^{\frac{1}{2}}}\right)\cos\left(\dfrac{\lambda^2}{4\|g\|}\right) + S\left(\dfrac{\lambda}{2^{\frac{1}{2}}\pi^{\frac{1}{2}}\|g\|^{\frac{1}{2}}}\right)\sin\left(\dfrac{\lambda^2}{4\|g\|}\right)\right]$
845.5	$\|p\|^{\frac{1}{2}}\sin(\lambda\|p\|^{\frac{1}{2}})e^{-\beta\|p\|}$ $\boxed{S}$ $\lambda = 0$: pair 635 $\boxed{L}$ $\beta \to 0$: pair 843	$\dfrac{i}{8\pi^{\frac{1}{2}}}\left[\dfrac{\lambda^2}{(\beta+ig)^{\frac{5}{2}}} - \dfrac{2}{(\beta+ig)^{\frac{3}{2}}}\right] \times \exp\left[-\dfrac{\lambda^2}{4(\beta+ig)}\right]\mathrm{erf}\left[\dfrac{i\lambda}{2(\beta+ig)^{\frac{1}{2}}}\right] + \dfrac{i}{8\pi^{\frac{1}{2}}}\left[\dfrac{\lambda^2}{(\beta-ig)^{\frac{5}{2}}} - \dfrac{2}{(\beta-ig)^{\frac{3}{2}}}\right] \times \exp\left[-\dfrac{\lambda^2}{4(\beta-ig)}\right]\mathrm{erf}\left[\dfrac{i\lambda}{2(\beta-ig)^{\frac{1}{2}}}\right] + \dfrac{\lambda(\beta^2-g^2)}{2\pi(\beta^2+g^2)^2}$
845.7	$\sin(\lambda\|p\|^{\frac{1}{2}})e^{-\beta\|p\|}$ $\boxed{S}$ $\lambda = 0$: pair 634.5 $\boxed{L}$ $\beta \to 0$: pair 845.9	$\dfrac{\lambda}{4\pi^{\frac{1}{2}}(\beta+ig)^{\frac{3}{2}}}\exp\left[-\dfrac{\lambda^2}{4(\beta+ig)}\right] + \dfrac{\lambda}{4\pi^{\frac{1}{2}}(\beta-ig)^{\frac{3}{2}}}\exp\left[-\dfrac{\lambda^2}{4(\beta-ig)}\right]$
845.9	$\sin(\lambda\|p\|^{\frac{1}{2}}) = \lim\limits_{\beta\to 0}[\sin(\lambda\|p\|^{\frac{1}{2}})e^{-\beta\|p\|}]$ $\boxed{S}$ $\lambda = 0$: pair 505.1	$\dfrac{\lambda}{2^{\frac{3}{2}}\pi^{\frac{1}{2}}\|g\|^{\frac{3}{2}}}\left[\sin\left(\dfrac{\lambda^2}{4\|g\|}\right) - \cos\left(\dfrac{\lambda^2}{4\|g\|}\right)\right]$
846	$\dfrac{1}{\|p\|^{\frac{1}{2}}}\sin(\lambda\|p\|^{\frac{1}{2}})e^{-\rho\|p\|}$ $\boxed{S}$ $\rho = 0$: pair 846.2 $\boxed{R}$ $\lambda = 0$: pair 632 $\boxed{O}$ $\lambda \neq R(\lambda)$, $R(\rho) = 0$	$\dfrac{1}{i2\pi^{\frac{1}{2}}(\rho+ig)^{\frac{1}{2}}}\exp\left[-\dfrac{\lambda^2}{4(\rho+ig)}\right] \times \mathrm{erf}\left[\dfrac{i\lambda}{2(\rho+ig)^{\frac{1}{2}}}\right] + \dfrac{1}{i2\pi^{\frac{1}{2}}(\rho-ig)^{\frac{1}{2}}}\exp\left[-\dfrac{\lambda^2}{4(\rho-ig)}\right] \times \mathrm{erf}\left[\dfrac{i\lambda}{2(\rho-ig)^{\frac{1}{2}}}\right]$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
846 2	$\frac{1}{\|p\|^{\frac12}}\sin(x\|p\|^{\frac12})$ $x = 0$	$\frac{2^{\frac12}}{\pi^{\frac12}\|g\|^{\frac12}}\left[C\left(\frac{x}{2^{\frac12}\pi^{\frac12}\|g\|^{\frac12}}\right)\sin\left(\frac{x^2}{4\|g\|}\right) - S\left(\frac{x}{2^{\frac12}\pi^{\frac12}\|g\|^{\frac12}}\right)\cos\left(\frac{x^2}{4\|g\|}\right)\right]$
847 0	$\frac{\|p\|^{\frac12}}{p}\exp(-\lambda\|p\|^{\frac12} - \rho\|p\|)$ $\lambda = 0$ pair 640 1 $\rho = 0$ pair 847 2 $R(\lambda) < 0 \quad R(\rho) = 0$	$\frac{i}{2\pi^{\frac12}(\rho + ig)^{\frac12}}\exp\left[\frac{\lambda^2}{4(\rho + ig)}\right] \times \operatorname{erfc}\left[\frac{\lambda}{2(\rho + ig)^{\frac12}}\right]$ $- \frac{i}{2\pi^{\frac12}(\rho - ig)^{\frac12}}\exp\left[\frac{\lambda^2}{4(\rho - ig)}\right] \times \operatorname{erfc}\left[\frac{\lambda}{2(\rho - ig)^{\frac12}}\right]$
847 2	$\frac{\|p\|^{\frac12}}{p}\exp(-\rho\|p\|^{\frac12})$ $\rho = 0$ pair 523 5	$\pm\frac{2^{\frac12}}{\pi^{\frac12}\|g\|^{\frac12}}\left\{\left[\tfrac12 - S\left(\frac{\rho}{2^{\frac12}\pi^{\frac12}\|g\|^{\frac12}}\right)\right] \times \sin\left(\frac{\rho^2}{4\|g\|}\right) + \left[\tfrac12 - C\left(\frac{\rho}{2^{\frac12}\pi^{\frac12}\|g\|^{\frac12}}\right)\right]\cos\left(\frac{\rho^2}{4\|g\|}\right)\right\}$ $0 < \pm g$
847 5	$\frac{\|p\|^{\frac12}}{p}\sin(\lambda\|p\|^{\frac12})e^{-\rho\|p\|}$ $\rho = 0$ pair 844 $\lambda = 0$ pair 638 1 $\lambda \mp R(\lambda) \quad R(\rho) = 0$	$\frac{1}{2\pi^{\frac12}(\rho + ig)^{\frac12}}\exp\left[-\frac{\lambda^2}{4(\rho + ig)}\right] \times \operatorname{erf}\left[\frac{i\lambda}{2(\rho + ig)^{\frac12}}\right]$ $- \frac{1}{2\pi^{\frac12}(\rho - ig)^{\frac12}}\exp\left[-\frac{\lambda^2}{4(\rho - ig)}\right] \times \operatorname{erf}\left[\frac{i\lambda}{2(\rho - ig)^{\frac12}}\right]$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
848.1	$\exp(-\lambda\lvert p\rvert^{1/2}-\rho\lvert p\rvert)$ Ⓢ $\lambda=0$: pair 632 Ⓢ $\rho=0$: pair 848.5 Ⓓ $R(\lambda)\leqq 0,\ R(\rho)=0$	$\dfrac{\rho}{\pi(\rho^2+g^2)}-\dfrac{\lambda}{4\pi^{1/2}(\rho+ig)^{3/2}}\exp\left[\dfrac{\lambda^2}{4(\rho+ig)}\right]\times\operatorname{erfc}\left[\dfrac{\lambda}{2(\rho+ig)^{1/2}}\right]-\dfrac{\lambda}{4\pi^{1/2}(\rho-ig)^{3/2}}\exp\left[\dfrac{\lambda^2}{4(\rho-ig)}\right]\times\operatorname{erfc}\left[\dfrac{\lambda}{2(\rho-ig)^{1/2}}\right]$
848.5	$\exp(-\alpha\lvert p\rvert^{1/2})$	$\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}\left\{\left[\tfrac{1}{2}-S\left(\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\right]\times\sin\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)+\left[\tfrac{1}{2}-C\left(\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\right]\cos\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)\right\}$
849.1	$\lvert p\rvert^{1/2}\exp(-\lambda\lvert p\rvert^{1/2}-\rho\lvert p\rvert)$ Ⓢ $\lambda=0$: pair 634.5 Ⓢ $\rho=0$: pair 849.5 Ⓓ $R(\lambda)\leqq 0,\ R(\rho)=0$	$\left[\dfrac{1}{4\pi^{1/2}(\rho+ig)^{3/2}}+\dfrac{\lambda^2}{8\pi^{1/2}(\rho+ig)^{5/2}}\right]\times\exp\left[\dfrac{\lambda^2}{4(\rho+ig)}\right]\operatorname{erfc}\left[\dfrac{\lambda}{2(\rho+ig)^{1/2}}\right]+\left[\dfrac{1}{4\pi^{1/2}(\rho-ig)^{3/2}}+\dfrac{\lambda^2}{8\pi^{1/2}(\rho-ig)^{5/2}}\right]\times\exp\left[\dfrac{\lambda^2}{4(\rho-ig)}\right]\operatorname{erfc}\left[\dfrac{\lambda}{2(\rho-ig)^{1/2}}\right]-\dfrac{\lambda(\rho^2-g^2)}{2\pi(\rho^2+g^2)^2}$
849.5	$\lvert p\rvert^{1/2}\exp(-\alpha\lvert p\rvert^{1/2})$	$\dfrac{\alpha}{2\pi g^2}-\dfrac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}\left[\tfrac{1}{2}-S\left(\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\right]\times\left[\sin\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)+\dfrac{\alpha^2}{2\lvert g\rvert}\cos\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)\right]-\dfrac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}\left[\tfrac{1}{2}-C\left(\dfrac{\alpha}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\right]\times\left[\cos\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)-\dfrac{\alpha^2}{2\lvert g\rvert}\sin\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)\right]$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
850 1	$\lvert p\rvert^{1/2}\cos(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}$ $\lambda = 0$ pair 634 5 $\beta \to 0$ pair 850 3	$\frac{1}{8\pi^{1/2}}\left[\frac{2}{(\beta+ig)^{3/2}} - \frac{\lambda^2}{(\beta+ig)^{5/2}}\right] \times \exp\left[-\frac{\lambda^2}{4(\beta+ig)}\right] + \frac{1}{8\pi^{1/2}}\left[\frac{2}{(\beta-ig)^{3/2}} - \frac{\lambda^2}{(\beta-ig)^{5/2}}\right] \times \exp\left[-\frac{\lambda^2}{4(\beta-ig)}\right]$
850 3	$\lvert p\rvert^{1/2}\cos(\lambda\lvert p\rvert^{1/2}) = \lim_{\beta\to 0}\left[\lvert p\rvert^{1/2}\cos(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}\right]$ $\lambda = 0$ pair 505 1	$\frac{1}{2^{5/2}\pi^{1/2}\lvert g\rvert^{3/2}}\left[\left(\frac{\lambda^2}{2\lvert g\rvert} - 1\right)\cos\left(\frac{\lambda^2}{4\lvert g\rvert}\right) + \left(\frac{\lambda^2}{2\lvert g\rvert} + 1\right)\sin\left(\frac{\lambda^2}{4\lvert g\rvert}\right)\right]$
851 1	$\frac{1}{\lvert p\rvert^{1/2}}\cos(\lambda\lvert p\rvert^{1/2})e^{-\rho\lvert p\rvert}$ $\lambda = 0$ pair 634 $\rho = 0$ pair 851 3 $\lambda \ddagger R(\lambda)$ $R(\rho) = 0$	$\frac{1}{2\pi^{1/2}(\rho+ig)^{1/2}}\exp\left[-\frac{\lambda^2}{4(\rho+ig)}\right] + \frac{1}{2\pi^{1/2}(\rho-ig)^{1/2}}\exp\left[-\frac{\lambda^2}{4(\rho-ig)}\right]$
851 3	$\frac{1}{\lvert p\rvert^{1/2}}\cos(x\lvert p\rvert^{1/2})$ $x = 0$ pair 523 1	$\frac{1}{2^{3/2}\pi^{1/2}\lvert g\rvert^{1/2}}\left[\cos\left(\frac{x^2}{4\lvert g\rvert}\right) + \sin\left(\frac{x^2}{4\lvert g\rvert}\right)\right]$
852 1	$\frac{\lvert p\rvert^{1/2}}{p}\cos(\lambda\lvert p\rvert^{1/2})e^{-\rho\lvert p\rvert}$ $\lambda = 0$ pair 640 1 $\rho = 0$ pair 852 3 $\lambda \ddagger R(\lambda)$ $R(\rho) = 0$	$\frac{i}{2\pi^{1/2}(\rho+ig)^{1/2}}\exp\left[-\frac{\lambda^2}{4(\rho+ig)}\right] - \frac{i}{2\pi^{1/2}(\rho-ig)^{1/2}}\exp\left[-\frac{\lambda^2}{4(\rho-ig)}\right]$
852 3	$\frac{\lvert p\rvert^{1/2}}{p}\cos(x\lvert p\rvert^{1/2})$ $x = 0$ pair 523 5	$\pm\frac{1}{2^{3/2}\pi^{1/2}\lvert g\rvert^{1/2}}\left[\cos\left(\frac{x^2}{4\lvert g\rvert}\right) - \sin\left(\frac{x^2}{4\lvert g\rvert}\right)\right] \quad 0 < \pm g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
853.1	$\dfrac{p}{\lvert p\rvert^{3/2}}\exp(-\lambda\lvert p\rvert^{1/2}-\rho\lvert p\rvert)$ S $\lambda=0$: pair 641.1 S $\rho=0$: pair 853.3 D $R(\lambda)\leqq 0,\ R(\rho)=0$	$\dfrac{1}{i8\pi^{1/2}}\left[\dfrac{\lambda^2}{(\rho+ig)^{3/2}}+\dfrac{2}{(\rho+ig)^{1/2}}\right]$ $\times\exp\left[\dfrac{\lambda^2}{4(\rho+ig)}\right]\operatorname{erfc}\left[\dfrac{\lambda}{2(\rho+ig)^{1/2}}\right]$ $-\dfrac{1}{i8\pi^{1/2}}\left[\dfrac{\lambda^2}{(\rho-ig)^{3/2}}+\dfrac{2}{(\rho-ig)^{1/2}}\right]$ $\times\exp\left[\dfrac{\lambda^2}{4(\rho-ig)}\right]\operatorname{erfc}\left[\dfrac{\lambda}{2(\rho-ig)^{1/2}}\right]$ $+\dfrac{\lambda\rho g}{\pi(\rho^2+g^2)^2}$
853.3	$\dfrac{p}{\lvert p\rvert^{3/2}}\exp(-\alpha\lvert p\rvert^{1/2})$	$\pm\dfrac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\left[\tfrac{1}{2}-C\left(\dfrac{\alpha}{2^{3/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\right]$ $\times\left[\sin\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)+\dfrac{\alpha^2}{2\lvert g\rvert}\cos\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)\right]$ $\mp\dfrac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\left[\tfrac{1}{2}-S\left(\dfrac{\alpha}{2^{3/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right)\right]$ $\times\left[\cos\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)-\dfrac{\alpha^2}{2\lvert g\rvert}\sin\left(\dfrac{\alpha^2}{4\lvert g\rvert}\right)\right],$ $0<\pm g$
854.1	$\dfrac{p}{\lvert p\rvert^{3/2}}\cos(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}$ S $\lambda=0$: pair 641.1 L $\beta\to 0$: pair 854.3	$\dfrac{i}{8\pi^{1/2}}\left[\dfrac{\lambda^2}{(\beta+ig)^{3/2}}-\dfrac{2}{(\beta+ig)^{1/2}}\right]$ $\times\exp\left[-\dfrac{\lambda^2}{4(\beta+ig)}\right]$ $-\dfrac{i}{8\pi^{1/2}}\left[\dfrac{\lambda^2}{(\beta-ig)^{3/2}}-\dfrac{2}{(\beta-ig)^{1/2}}\right]$ $\times\exp\left[-\dfrac{\lambda^2}{4(\beta-ig)}\right]$
854.3	$\dfrac{p}{\lvert p\rvert^{3/2}}\cos(\lambda\lvert p\rvert^{1/2})$ $=\lim\limits_{\beta\to 0}\left[\dfrac{p}{\lvert p\rvert^{3/2}}\cos(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}\right]$	$\mp\dfrac{1}{2^{3/2}\pi^{1/2}\lvert g\rvert^{1/2}}\left[\left(1-\dfrac{\lambda^2}{2\lvert g\rvert}\right)\sin\left(\dfrac{\lambda^2}{4\lvert g\rvert}\right)\right.$ $\left.+\left(1+\dfrac{\lambda^2}{2\lvert g\rvert}\right)\cos\left(\dfrac{\lambda^2}{4\lvert g\rvert}\right)\right],$ $0<\pm g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
855 1	$\frac{p}{\lvert p\rvert^{1/2}}\sin(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}$ S $\lambda = 0$ pair 632 7 □ $\beta \to 0$ pair 855 3	$\begin{aligned} &\frac{1}{8\pi^{1/2}}\left[\frac{\lambda^2}{(\beta+ig)^{5/2}} - \frac{2}{(\beta+ig)^{3/2}}\right] \\ &\quad \times \exp\left[-\frac{\lambda^2}{4(\beta+ig)}\right]\operatorname{erf}\left[\frac{i\lambda}{2(\beta+ig)^{1/2}}\right] \\ &\quad - \frac{1}{8\pi^{1/2}}\left[\frac{\lambda^2}{(\beta-ig)^{5/2}} - \frac{2}{(\beta-ig)^{3/2}}\right] \\ &\quad \times \exp\left[-\frac{\lambda^2}{4(\beta-ig)}\right]\operatorname{erf}\left[\frac{i\lambda}{2(\beta-ig)^{1/2}}\right] \\ &\qquad\qquad - \frac{\beta\lambda g}{\pi(\beta^2+g^2)^2} \end{aligned}$
855 3	$\frac{p}{\lvert p\rvert^{1/2}}\sin(\lambda\lvert p\rvert^{1/2})$ $= \lim_{\beta\to 0}\left[\frac{p}{\lvert p\rvert^{1/2}}\sin(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}\right]$	$\begin{aligned} &\mp\frac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}C\left(\frac{\lambda}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right) \\ &\quad \times\left[\sin\left(\frac{\lambda^2}{4\lvert g\rvert}\right) + \frac{\lambda^2}{2\lvert g\rvert}\cos\left(\frac{\lambda^2}{4\lvert g\rvert}\right)\right] \\ &\quad \pm\frac{1}{2^{1/2}\pi^{1/2}\lvert g\rvert^{3/2}}S\left(\frac{\lambda}{2^{1/2}\pi^{1/2}\lvert g\rvert^{1/2}}\right) \\ &\quad \times\left[\cos\left(\frac{\lambda^2}{4\lvert g\rvert}\right) - \frac{\lambda^2}{2\lvert g\rvert}\sin\left(\frac{\lambda^2}{4\lvert g\rvert}\right)\right] \\ &\qquad\qquad 0 < \pm g \end{aligned}$
856 1	$\frac{\lvert p\rvert}{p}\cos(\lambda\lvert p\rvert^{1/2})e^{-\beta\lvert p\rvert}$ S $\lambda = 0$ pair 638 1 □ $\beta \to 0$ pair 856 3	$\begin{aligned} &\frac{g}{\pi(\beta^2+g^2)} \\ &\quad - \frac{\lambda}{4\pi^{1/2}(\beta+ig)^{3/2}}\exp\left[-\frac{\lambda^2}{4(\beta+ig)}\right] \\ &\quad \times\operatorname{erf}\left[\frac{i\lambda}{2(\beta+ig)^{1/2}}\right] \\ &\quad + \frac{\lambda}{4\pi^{1/2}(\beta-ig)^{3/2}}\exp\left[-\frac{\lambda^2}{4(\beta-ig)}\right] \\ &\qquad\qquad \times\operatorname{erf}\left[\frac{i\lambda}{2(\beta-ig)^{1/2}}\right] \end{aligned}$

No.	Coefficient $F(f)$	Coefficient $G(g)$
856.3	$\dfrac{\lvert p\rvert}{p}\cos(\lambda\lvert p\rvert^{\frac12})$ $= \left[\lim_{\beta\to 0}\dfrac{\lvert p\rvert}{p}\cos(\lambda\lvert p\rvert^{\frac12})e^{-\beta\lvert p\rvert}\right]$	$\dfrac{1}{\pi g} \pm \dfrac{\lambda}{2^{\frac12}\pi^{\frac12}\lvert g\rvert^{\frac32}}\Big[S\Big(\dfrac{\lambda}{2^{\frac12}\pi^{\frac12}\lvert g\rvert^{\frac12}}\Big) \times \cos\Big(\dfrac{\lambda^2}{4\lvert g\rvert}\Big) - C\Big(\dfrac{\lambda}{2^{\frac12}\pi^{\frac12}\lvert g\rvert^{\frac12}}\Big) \times \sin\Big(\dfrac{\lambda^2}{4\lvert g\rvert}\Big)\Big],\quad 0 < \pm g$
857.1	$\dfrac{\lvert p\rvert}{p}\sin(\lambda\lvert p\rvert^{\frac12})e^{-\beta\lvert p\rvert}$ $\boxed{S}$ $\lambda = 0$: pair 641.1 $\boxed{L}$ $\beta \to 0$: pair 857.3	$\dfrac{i\lambda}{4\pi^{\frac12}(\beta+ig)^{\frac32}}\exp\Big[-\dfrac{\lambda^2}{4(\beta+ig)}\Big] - \dfrac{i\lambda}{4\pi^{\frac12}(\beta-ig)^{\frac32}}\exp\Big[-\dfrac{\lambda^2}{4(\beta-ig)}\Big]$
857.3	$\dfrac{\lvert p\rvert}{p}\sin(\lambda\lvert p\rvert^{\frac12})$ $= \lim_{\beta\to 0}\left[\dfrac{\lvert p\rvert}{p}\sin(\lambda\lvert p\rvert^{\frac12})e^{-\beta\lvert p\rvert}\right]$	$\pm\dfrac{\lambda}{2^{\frac32}\pi^{\frac12}\lvert g\rvert^{\frac32}}\Big[\cos\Big(\dfrac{\lambda^2}{4\lvert g\rvert}\Big) + \sin\Big(\dfrac{\lambda^2}{4\lvert g\rvert}\Big)\Big],\quad 0 < \pm g$
859.1	$\exp\{\mu[(p+\rho)^{\frac12} - (p+\sigma)^{\frac12}]^2\} - 1$ $\boxed{S}$ $\mu = 0$: pair 559.2 $\boxed{B}$ $\mu = \infty$: pair 654.2 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 817 $\boxed{R}$ $\rho = \sigma$: pair 438	$\dfrac{\mu(\rho-\sigma)e^{-\frac12(\rho+\sigma)g}I_1[\frac12(\rho-\sigma)g^{\frac12}(g+4\mu)^{\frac12}]}{g^{\frac12}(g+4\mu)^{\frac12}},\quad 0 < g$
860.0	$\dfrac{\exp[-x(p+\rho)^{\frac12}(p+\sigma)^{\frac12}]}{(p+\rho)^{\frac12}(p+\sigma)^{\frac12}}$ $\boxed{S}$ $x = 0$: pair 555 $\boxed{S}$ $\rho = 0$ or $\sigma = 0$: pair 861 $\boxed{S}$ $\rho = -\sigma$: pair 866 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 823 $\boxed{R}$ $\rho = \sigma$: pair 604	$e^{-\frac12(\rho+\sigma)g}I_0[\frac12(\rho-\sigma)(g^2-x^2)^{\frac12}],\quad x < g$
860.5	$\dfrac{\exp[\mu p - \mu(p+\rho)^{\frac12}(p+\sigma)^{\frac12}]}{(p+\rho)^{\frac12}(p+\sigma)^{\frac12}}$ $\boxed{S}$ $\mu = 0$: pair 555 $\boxed{S}$ $\rho = -\sigma$: pair 860.6 $\boxed{B}$ $\mu = \infty$: pair 655.1 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 823 $\boxed{R}$ $\rho = \sigma$: pair 438	$e^{-\frac12(\rho+\sigma)(g+\mu)}I_0[\frac12(\rho-\sigma)g^{\frac12}(g+2\mu)^{\frac12}],\quad 0 < g$

No	Coefficient $F(f)$	Coefficient $G(g)$
860.6	$\dfrac{\exp[\mu p - \mu(p^2 + x^2)^{\frac{1}{2}}]}{(p^2 + x^2)^{\frac{1}{2}}}$ $\mu = 0$ pair 557 $\mu = \infty$ pair 655.2 $x = 0$	$I_0[xg^{\frac{1}{2}}(g + 2\mu)^{\frac{1}{2}}]$ $\quad 0 < g$
861	$\dfrac{\exp[-xp^{\frac{1}{2}}(p + \rho)^{\frac{1}{2}}]}{p^{\frac{1}{2}}(p + \rho)^{\frac{1}{2}}}$ $x = 0$ pair 563.4 $\rho = \infty$ pair 807 $\rho = 0$	$e^{-\frac{1}{2}\rho g}I_0[\frac{1}{2}\rho(g^2 - x^2)^{\frac{1}{2}}]$ $\quad x < g$
862.1	$\left(\dfrac{p}{p + \rho}\right)^{\frac{1}{2}}\exp[-xp^{\frac{1}{2}}(p + \rho)^{\frac{1}{2}}] - \exp(-xp - \frac{1}{2}x\rho)$ $x = 0$ pair 553.5 $\rho = \infty$ pair 806 $\rho = 0$	$\frac{1}{2}\rho e^{-\frac{1}{2}\rho g}\left\{\dfrac{g}{(g^2 - x^2)^{\frac{1}{2}}}I_1[\frac{1}{2}\rho(g^2 - x^2)^{\frac{1}{2}}] - I_0[\frac{1}{2}\rho(g^2 - x^2)^{\frac{1}{2}}]\right\}$ $\quad x < g$
863.1	$\exp[-x(p + \rho)^{\frac{1}{2}}(p + \sigma)^{\frac{1}{2}}] - \exp[-xp - \frac{1}{2}x(\rho + \sigma)]$ $x = 0$ pair 559.2 $\rho = -\sigma$ pair 865.1 $\rho = \infty$ or $\sigma = \infty$ pair 817 $\rho = \sigma$ pair 604	$\dfrac{x(\rho - \sigma)e^{-\frac{1}{2}(\rho+\sigma)g}I_1[\frac{1}{2}(\rho - \sigma)(g^2 - x^2)^{\frac{1}{2}}]}{2(g^2 - x^2)^{\frac{1}{2}}},$ $\quad x < g$
864.1	$\left(\dfrac{p + \rho}{p + \sigma}\right)^{\frac{1}{2}}\exp[-x(p + \rho)^{\frac{1}{2}}(p + \sigma)^{\frac{1}{2}}] - \exp[-xp - \frac{1}{2}x(\rho + \sigma)]$ $x = 0$ pair 561.0 $\rho = 0$ pair 862.1 $\rho = \infty$ pair 823 $\sigma = \infty$ pair 820 $\rho = \sigma$ pair 604	$\frac{1}{2}(\rho - \sigma)e^{-\frac{1}{2}(\rho+\sigma)g} \times \left\{\dfrac{g}{(g^2 - x^2)^{\frac{1}{2}}}I_1[\frac{1}{2}(\rho - \sigma)(g^2 - x^2)^{\frac{1}{2}}] + I_0[\frac{1}{2}(\rho - \sigma)(g^2 - x^2)^{\frac{1}{2}}]\right\}$, $\quad x < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
864.5	$\dfrac{\exp[-x(p+\rho)^{1/2}(p+\sigma)^{1/2}]}{(p+\rho)^{1/2}(p+\sigma)^{1/2}[(p+\rho)^{1/2}+(p+\sigma)^{1/2}]^2}$ $\boxed{B}$ $\rho=\infty$ or $\sigma=\infty$: pair 823 $\boxed{D}$ $R(\rho)=0,\ \rho=\sigma$	$\dfrac{1}{\rho-\sigma}\left(\dfrac{g-x}{g+x}\right)^{1/2} e^{-\frac{1}{2}(\rho+\sigma)g}$ $\times I_1[\tfrac{1}{2}(\rho-\sigma)(g^2-x^2)^{1/2}],\quad x<g$
865.1	$\exp[-x(p^2+y^2)^{1/2}]-e^{-xp}$ $\boxed{S}$ $x=0$: pair 556.1 $\boxed{D}$ $y=0$	$-\dfrac{xyJ_1[y(g^2-x^2)^{1/2}]}{(g^2-x^2)^{1/2}},\quad x<g$
866	$\dfrac{\exp[-x(p^2+y^2)^{1/2}]}{(p^2+y^2)^{1/2}}$ $\boxed{S}$ $x=0$: pair 557 $\boxed{D}$ $y=0$	$J_0[y(g^2-x^2)^{1/2}],\quad x<g$
867	$\exp[-\alpha(\rho^2-p^2)^{1/2}]$ $\boxed{S}$ $\rho=0$: pair 632 $\boxed{B}$ $\alpha=\infty$ and $\rho=\infty$: pair 710.0	$\dfrac{\alpha\rho K_1[\rho(g^2+\alpha^2)^{1/2}]}{\pi(g^2+\alpha^2)^{1/2}}$
867.5	$\dfrac{1+\sigma(\beta^2-p^2)^{1/2}}{(\beta^2-p^2)^{1/2}}\exp[-\sigma(\beta^2-p^2)^{1/2}]$ $\boxed{S}$ $\sigma=0$: pair 558 5 $\boxed{B}$ $\beta=\infty$ and $\sigma=\infty$: pair 710.0	$\dfrac{1}{\pi\beta}(g^2+\sigma^2)^{1/2}K_1[\beta(g^2+\sigma^2)^{1/2}]$
868	$\dfrac{\exp[-\sigma(\rho^2-p^2)^{1/2}]}{(\rho^2-p^2)^{1/2}}$ $\boxed{S}$ $\sigma=0$: pair 558 $\boxed{B}$ $\rho=\infty$ and $\sigma=\infty$: pair 710.0 $\boxed{D}$ $\rho=0$.	$\dfrac{1}{\pi}K_0[\rho(g^2+\sigma^2)^{1/2}]$
869	$\dfrac{\exp[-x(p^2+y^2)^{1/2}]}{(p^2+y^2)^{1/2}[(p^2+y^2)^{1/2}+p]^{\alpha-1}}$ $\boxed{S}$ $\alpha=1$: pair 866 $\boxed{S}$ $x=0$: pair 575.2 $\boxed{R}$ $y=0$: pair 606.1	$\dfrac{1}{y^{\alpha-1}}\left(\dfrac{g-x}{g+x}\right)^{\frac{1}{2}\alpha-\frac{1}{2}} J_{\alpha-1}[y(g^2-x^2)^{1/2}],$ $x<g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
870 1 Key	$\dfrac{\exp\{\mu[(p+\rho)^{\frac{1}{2}}-(p+\sigma)^{\frac{1}{2}}]^{2}\}}{(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}[(p+\rho)^{\frac{1}{2}}+(p+\sigma)^{\frac{1}{2}}]^{2\nu-2}}$	$\dfrac{g^{\frac{1}{2}\nu-\frac{1}{2}}e^{-\frac{1}{2}(\rho+\sigma)g}I_{\nu-1}[\frac{1}{2}(\rho-\sigma)g^{\frac{1}{2}}(g+4\mu)^{\frac{1}{2}}]}{(\rho-\sigma)^{\nu-1}(g+4\mu)^{\frac{1}{2}\nu-\frac{1}{2}}}$ $\quad 0 < g$
	$\nu = 1$ pair 860 5	
	$\mu = 0$ pair 575 1	
	$\rho = -\sigma$ pair 870 3	
	$\mu = \infty$ pair 650	
	$\rho = \infty$ or $\sigma = \infty$ pair 823	
	$\rho = \sigma$ pair 524 ?	
870 3	$\dfrac{\exp[\mu p-\mu(p^{2}+x^{2})^{\frac{1}{2}}]}{(p^{2}+x^{2})^{\frac{1}{2}}[(p^{2}+x^{2})^{\frac{1}{2}}+p]^{\nu-1}}$	$\dfrac{g^{\frac{1}{2}\nu-\frac{1}{2}}J_{\nu-1}[xg^{\frac{1}{2}}(g+2\mu)^{\frac{1}{2}}]}{x^{\nu-1}(g+2\mu)^{\frac{1}{2}\nu-\frac{1}{2}}},$ $\quad 0 < g$
	$\nu = 1$ pair 860 6	
	$\mu = 0$ pair 575 2	
	$\mu = \infty$ pair 650 4	
	$x = 0$ pair 521	
870 5	$\dfrac{\exp[-x(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}[(p+\rho)^{\frac{1}{2}}+(p+\sigma)^{\frac{1}{2}}]^{2\nu}}$	$\dfrac{1}{(\rho-\sigma)^{\nu-1}}\left(\dfrac{g-x}{g+x}\right)^{\frac{1}{2}\nu-\frac{1}{2}}e^{-\frac{1}{2}(\rho+\sigma)g}$ $\times I_{\nu-1}[\frac{1}{2}(\rho-\sigma)(g^{2}-x^{2})^{\frac{1}{2}}]$ $\quad x < g$
	$\nu = 1, 2$ pairs 860 0, 864 5	
	$x = 0$ pair 575 1	
	$\rho = -\sigma$ pair 869	
	$\rho = \infty$ or $\sigma = \infty$ pair 823	
	$\rho = \sigma$ pair 605 1	
870 8	$\left(\dfrac{p+\rho}{p+\sigma}\right)^{\frac{1}{2}}\dfrac{\exp[-x(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}]}{[(p+\rho)^{\frac{1}{2}}+(p+\sigma)^{\frac{1}{2}}]^{2\nu}}$	$\dfrac{1}{4(\rho-\sigma)^{\nu-1}}\left(\dfrac{g-x}{g+x}\right)^{\frac{1}{2}\nu-\frac{1}{2}}e^{-\frac{1}{2}(\rho+\sigma)g}$ $\times\Big\{I_{\nu-1}[\frac{1}{2}(\rho-\sigma)(g^{2}-x^{2})^{\frac{1}{2}}]$ $+2\left(\dfrac{g-x}{g+x}\right)^{\frac{1}{2}}I_{\nu}[\frac{1}{2}(\rho-\sigma)(g^{2}-x^{2})^{\frac{1}{2}}]$ $+\left(\dfrac{g-x}{g+x}\right)I_{\nu+1}[\frac{1}{2}(\rho-\sigma)(g^{2}-x^{2})^{\frac{1}{2}}]\Big\}$ $\quad x < g$
	$x = 0$ pair 573 5	
	$\rho = \infty$ pair 823	
	$\sigma = \infty$ pair 820	
	$\rho = \sigma$ pair 605 1	
871 2	$\cosh[a(p^{2}-\lambda^{2})^{\frac{1}{2}}]-\cosh ap$	$-\dfrac{a\lambda J_{1}[\lambda(a^{2}-g^{2})^{\frac{1}{2}}]}{2(a^{2}-g^{2})^{\frac{1}{2}}}$ $\quad \lvert g\rvert < a$
	$\lambda = 0$ pair 622	

No.	Coefficient $F(f)$	Coefficient $G(g)$
871.3	$\cosh[c(p+\lambda)^{1/2}(p+\mu)^{1/2}] - \cosh[cp + \frac{1}{2}c(\lambda+\mu)]$ $\boxed{S}$ $\lambda = \mu$: pair 622.1 $\boxed{S}$ $\lambda = -\mu$: pair 871.2	$\dfrac{-c(\lambda-\mu)e^{-\frac{1}{2}(\lambda+\mu)g}J_1[\frac{1}{2}(\lambda-\mu)(c^2-g^2)^{1/2}]}{4(c^2-g^2)^{1/2}}$, $\quad \|g\| < c$
871.5	$\dfrac{\cos[x(\rho^2-p^2)^{1/2}]}{(\rho^2-p^2)^{1/2}}$ $\boxed{S}$ $x = 0$: pair 558 $\boxed{B}$ $x = \infty$ and $\rho = \infty$: pair 710.0 $\boxed{D}$ $\rho = 0$	$\begin{cases} -\frac{1}{2}Y_0[\rho(x^2-g^2)^{1/2}], & \|g\| < \|x\| \\ \frac{1}{\pi}K_0[\rho(g^2-x^2)^{1/2}], & \|x\| < \|g\| \end{cases}$
872	$\dfrac{\sin[a(\lambda^2-p^2)^{1/2}]}{(\lambda^2-p^2)^{1/2}}$ $\boxed{A}$ pair 872.1 $\boxed{S}$ $\lambda = 0$: pair 622	$\frac{1}{2}J_0[\lambda(a^2-g^2)^{1/2}]$, $\quad \|g\| < a$
872.1	$\dfrac{\sinh[a(p^2+\lambda^2)^{1/2}]}{(p^2+\lambda^2)^{1/2}}$ $\boxed{A}$ pair 872 $\boxed{S}$ $\lambda = 0$: pair 622	$\frac{1}{2}I_0[\lambda(a^2-g^2)^{1/2}]$, $\quad \|g\| < a$
872.2	$\dfrac{\sinh[a(p+\lambda)^{1/2}(p+\mu)^{1/2}]}{(p+\lambda)^{1/2}(p+\mu)^{1/2}}$ $\boxed{S}$ $\lambda = \mu$: pair 622.1 $\boxed{S}$ $\lambda = -\mu$: pair 872.1	$\frac{1}{2}e^{-\frac{1}{2}(\lambda+\mu)g}J_0[\frac{1}{2}(\lambda-\mu)(a^2-g^2)^{1/2}]$, $\quad \|g\| < a$
881	$\tan^{-1}\left(\dfrac{\lambda}{p+\rho}\right)$ $\boxed{A}$ pair 894 $\boxed{S}$ $\rho = 0$: pair 882.1 $\boxed{R}$ $\lambda = 0$: pair 438 $\boxed{D}$ $R(\rho) < \|I(\lambda)\|$	$\dfrac{1}{g}e^{-\rho g}\sin\lambda g$, $\quad 0 < g$
882.1	$\tan^{-1}\left(\dfrac{x}{p}\right)$ $\boxed{D}$ $x = 0$	$\dfrac{1}{g}\sin xg$, $\quad 0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
891	$\dfrac{\log(p+\rho)}{(p+\rho)^{\alpha}}$ $\alpha = 1$ pair 893 1 $\rho = 0$ pair 892 $1 \leqq R(\alpha)$ $R(\rho) = 0$	$\dfrac{\psi(\alpha) - \log g}{\Gamma(\alpha)} g^{\alpha-1} e^{-\rho g}$,	$0 < g$
891 5	$\dfrac{\log(p+\rho)}{(p+\rho)^{\alpha-1}} - \dfrac{\log(p+\sigma)}{(p+\sigma)^{\alpha-1}}$ $\alpha = 1$ pair 894 $\rho = \infty$ or $\sigma = \infty$ pair 891 $1 \leqq R(\alpha)$ $R(\rho) = 0$ $\rho \neq \sigma$ $2 \leqq R(\alpha)$ $R(\rho) = 0$ or $R(\sigma) = 0$	$\dfrac{\psi(\alpha-1) - \log g}{\Gamma(\alpha-1)} g^{\alpha-2}(e^{-\rho g} - e^{-\sigma g})$	$0 < g$
892	$\dfrac{\log p}{p^{\alpha}}$ $1 \leqq R(\alpha)$	$\dfrac{\psi(\alpha) - \log g}{\Gamma(\alpha)} g^{\alpha-1}$,	$0 < g$
893	$\dfrac{\log p}{p} = \lim\limits_{\beta \to 0}\left[\dfrac{\log(p+\beta)}{p+\beta}\right]$	$\psi(1) - \log g$	$0 < g$
893 1	$\dfrac{\log(p+\beta)}{p+\beta}$ $\beta \to 0$ pair 893	$[\psi(1) - \log g]e^{-\beta g}$,	$0 < g$
894	$\log\left(\dfrac{p+\rho}{p+\sigma}\right)$ pair 881 $\rho = 0$ or $\sigma = 0$ pair 894 2 $\rho = -\sigma$ pair 882 1 $\rho = \sigma$ pair 438	$\dfrac{1}{g}(e^{-\sigma g} - e^{-\rho g})$,	$0 < g$
894 2	$\log\left(\dfrac{p+\rho}{p}\right)$ $\rho = 0$	$\dfrac{1}{g}(1 - e^{-\rho g})$	$0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
	Part 9. Other Transcendental Functions of f		
902.1	$\frac{1}{p^{1/2}}\exp\left(\frac{1}{\sigma^2 p}\right)\operatorname{erfc}\left(\frac{1}{\sigma p^{1/2}}\right)$ $\boxed{\mathrm{B}}$ $\sigma = \infty$: pair 522 $\boxed{\mathrm{O}}$ $\sigma = 0$	$\frac{1}{(\pi g)^{1/2}}\exp\left(-\frac{2g^{1/2}}{\sigma}\right)$,	$0 < g$
902.5	$\frac{1}{(p+\rho)^{1/2}}\exp\left[\frac{1}{\lambda(p+\rho)}\right]$ $\times\operatorname{erfc}\left[\frac{1}{\lambda^{1/2}(p+\rho)^{1/2}}\right]-\frac{\lambda^{1/2}}{\pi^{1/2}(p+\rho)}$ $\boxed{\mathrm{S}}$ $\rho = 0$: pair 902.6 $\boxed{\mathrm{B}}$ $\lambda = \infty$: pair 438 $\boxed{\mathrm{O}}$ $\lambda = 0$ $\boxed{\mathrm{O}}$ $R(\lambda^{1/2}) < 0$ [$R(\lambda^{1/2}) \leqq 0$ for transposed pair], $R(\rho) = 0$	$-\frac{\lambda^{1/2}}{\pi^{1/2}}\exp\left(-\rho g-\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
902.6	$\frac{1}{p^{1/2}}\exp\left(\frac{1}{\sigma^2 p}\right)\operatorname{erfc}\left(\frac{1}{\sigma p^{1/2}}\right)-\frac{\sigma}{\pi^{1/2}p}$ $\boxed{\mathrm{O}}$ $\sigma = 0$ $\boxed{\mathrm{O}}$ $R(\sigma) = 0$ for transposed pair	$-\frac{\sigma}{\pi^{1/2}}\exp\left(-\frac{2g^{1/2}}{\sigma}\right)$,	$0 < g$
902.8	$\left[\frac{1}{(p+\rho)^{1/2}}+\frac{\lambda}{2(p+\rho)^{3/2}}\right]\exp\left[\frac{1}{\lambda(p+\rho)}\right]$ $\times\operatorname{erfc}\left[\frac{1}{\lambda^{1/2}(p+\rho)^{1/2}}\right]-\frac{\lambda^{1/2}}{\pi^{1/2}(p+\rho)^2}$ $\boxed{\mathrm{S}}$ $\rho = 0$: pair 902.9 $\boxed{\mathrm{B}}$ $\lambda = \infty$: pair 529 $\boxed{\mathrm{O}}$ $\lambda = 0$ $\boxed{\mathrm{O}}$ $R(\lambda^{1/2}) \leqq 0$, $R(\rho) = 0$	$\frac{\lambda g^{1/2}}{\pi^{1/2}}\exp\left(-\rho g-\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
902.9	$\left(\frac{1}{p^{1/2}}+\frac{\gamma^2}{2p^{3/2}}\right)\exp\left(\frac{1}{\gamma^2 p}\right)\operatorname{erfc}\left(\frac{1}{\gamma p^{1/2}}\right)$ $-\frac{\gamma}{\pi^{1/2}p^2}$	$\frac{\gamma^2 g^{1/2}}{\pi^{1/2}}\exp\left(-\frac{2g^{1/2}}{\gamma}\right)$,	$0 < g$
903.0	$\exp[\sigma(p+\lambda)^2]\operatorname{erfc}[\sigma^{1/2}(p+\lambda)]$ $\boxed{\mathrm{B}}$ $\sigma = \infty$: pair 438 $\boxed{\mathrm{O}}$ $\sigma = 0$ $\boxed{\mathrm{O}}$ $R(\lambda) < 0$, $R(\sigma) = 0$	$\frac{1}{(\pi\sigma)^{1/2}}\exp\left(-\lambda g-\frac{g^2}{4\sigma}\right)$,	$0 < g$

No	Coefficient $F(f)$	Coefficient $G(g)$	
903 1	$\frac{1}{p+\beta}\exp[\sigma(p+\beta)^2]\operatorname{erfc}[\sigma^{1/2}(p+\beta)]$ ⊠ $\sigma = 0$ pair 438 ⊠ $\sigma = \infty$ pair 442	$e^{-\beta g}\operatorname{erf}\left(\frac{g}{2\sigma^{1/2}}\right)$,	$0 < g$
903 3	$(p+\lambda)\exp[\sigma(p+\lambda)^2]\operatorname{erfc}[\sigma^{1/2}(p+\lambda)] - \frac{1}{(\pi\sigma)^{1/2}}$ ⊠ $\sigma = \infty$ pair 442 ⊠ $\sigma = 0$ ⊠ $R(\lambda) \leqq 0$ $R(\sigma) = 0$	$-\frac{g}{2\pi^{1/2}\sigma^{3/2}}\exp\left(-\lambda g - \frac{g^2}{4\sigma}\right)$,	$0 < g$
903 4	$\|p\|\exp(-\beta^2p^2)\operatorname{erfc}(\beta^{1/2}\|p\|) - \frac{1}{\pi^{1/2}\beta^{1/2}}$ ⊠ $\beta = 1/(4\pi)^{1/2}$ pair 903 45	$\frac{\|g\|}{4\pi^{1/2}\beta^{3}}\exp\left(\frac{g^2}{4\beta^2}\right)\operatorname{erfc}\left(\frac{\|g\|}{2\beta^{1/2}}\right) - \frac{1}{2\pi\beta^2}$	
903 45	$\|f\|\exp(\pi f^2)\operatorname{erfc}(\pi^{1/2}\|f\|) - \frac{1}{\pi}$	$\|g\|\exp(\pi g^2)\operatorname{erfc}(\pi^{1/2}\|g\|) - \frac{1}{\pi}$	
903 5	$\exp[\sigma(p+\beta)^2]\operatorname{erfc}[\sigma^{1/2}(p+\beta)] - \frac{1}{(\pi\sigma)^{1/2}(p+\beta)}$ ⊠ $\sigma = 0$ pair 438 ⊠ $\sigma = \infty$ pair 450	$\frac{1}{(\pi\sigma)^{1/2}}e^{-\beta g}\left[\exp\left(-\frac{g^2}{4\sigma}\right) - 1\right]$,	$0 < g$
904 1	$\frac{1}{(p+\rho)^{1/2}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times \operatorname{erf}\left[\frac{1}{\lambda^{1/2}(p+\rho)^{1/2}}\right]$ ⊠ $\lambda = \infty$ pair 438 ⊠ $\lambda = 0$ ⊠ $\lambda \neq -\|\lambda\|$, $R(\rho) = 0$	$\frac{1}{(\pi g)^{1/2}}e^{-\rho g}\sinh\left(\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$
904 2	$\operatorname{erf}\left[\frac{1}{\lambda^{1/2}(p+\rho)^{1/2}}\right]$ ⊠ $\rho = 0$ pair 904 4 ⊠ $\lambda = \infty$ pair 526 ⊠ $\lambda = 0$ ⊠ $\lambda \neq \|\lambda\|$, $R(\rho) = 0$	$\frac{1}{\pi g}e^{-\rho g}\sin\left(\frac{2g^{1/2}}{\lambda^{1/2}}\right)$,	$0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
904.31	$\frac{1}{p^{\frac{1}{2}}}\exp\left(\frac{1}{cp}\right)\operatorname{erf}\left(\frac{1}{c^{\frac{1}{2}}p^{\frac{1}{2}}}\right)$	$-\frac{1}{(\pi g)^{\frac{1}{2}}}\sinh\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$g < 0$
904.4	$\operatorname{erf}\left(\frac{1}{c^{\frac{1}{2}}p^{\frac{1}{2}}}\right)$ $\boxed{B}$ $c = \infty$: pair 522	$\frac{1}{\pi g}\sin\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$0 < g$
904.5	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times \operatorname{erf}\left[\frac{1}{\lambda^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}\right] + \frac{\lambda^{\frac{1}{2}}}{\pi^{\frac{1}{2}}(p+\rho)}$ $\boxed{B}$ $\lambda = \infty$: pair 438 $\boxed{O}$ $\lambda = 0$ $\boxed{O}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$ $\boxed{O}$ $R(\rho) = 0$ for transposed pair	$\frac{\lambda^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}e^{-\rho g}\cosh\left(\frac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
904.7	$\frac{1}{p^{\frac{1}{2}}}\exp\left(\frac{1}{cp}\right)\operatorname{erf}\left(\frac{1}{c^{\frac{1}{2}}p^{\frac{1}{2}}}\right) + \frac{c^{\frac{1}{2}}}{\pi^{\frac{1}{2}}p}$ $\boxed{F}$ No transposed pair	$-\frac{c^{\frac{1}{2}}}{\pi^{\frac{1}{2}}}\cosh\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$g < 0$
905.0	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times \operatorname{erfc}\left[\frac{1}{\lambda^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}\right]$ $\boxed{S}$ $\rho = 0$: pair 902.1 $\boxed{B}$ $\lambda = \infty$: pair 526 $\boxed{O}$ $\lambda = 0$ $\boxed{O}$ $R(\lambda^{\frac{1}{2}}) < 0$, $R(\rho) = 0$	$\frac{1}{(\pi g)^{\frac{1}{2}}}\exp\left(-\rho g - \frac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
906.1	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp(\sigma^2 p)\operatorname{erfc}[\sigma(p+\rho)^{\frac{1}{2}}]$ $\boxed{S}$ $\sigma = 0$: pair 526 $\boxed{B}$ $\sigma = \infty$: pair 438	$\frac{\exp[-\rho(g+\sigma^2)]}{\pi^{\frac{1}{2}}(g+\sigma^2)^{\frac{1}{2}}}$,	$0 < g$
906.3	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\operatorname{erfc}[x^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]$ $\boxed{S}$ $x = 0$: pair 526	$\frac{1}{(\pi g)^{\frac{1}{2}}}e^{-\rho g}$	$x < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
907 1	$\exp(\gamma^2 p)\operatorname{erfc}[\gamma(p+\rho)^{\frac{1}{2}}]$ E $\gamma = \infty$ pair 526	$\dfrac{\gamma\exp[-\rho(g+\gamma^2)]}{\pi g^{\frac{1}{2}}(g+\gamma^2)}$,	$0 < g$
907 3	$\operatorname{erfc}[a^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]$	$\dfrac{a^{\frac{1}{2}}e^{-\rho g}}{\pi g(g-a)^{\frac{1}{2}}}$	$a < g$
907 5	$\operatorname{erfc}[r^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}] - \dfrac{e^{-r(p+\rho)}}{(\pi r)^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}$ S $r = 0$ pair 526	$-\dfrac{e^{-\rho g}(g-r)^{\frac{1}{2}}}{\pi r^{\frac{1}{2}} g}$,	$r < g$
907 7	$\dfrac{\operatorname{erf}[a^{\frac{1}{2}}(p+\lambda)^{\frac{1}{2}}]}{(p+\lambda)^{\frac{1}{2}}}$ E $a = \infty$ pair 526	$\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{-\lambda g}$	$0 < g < a$
907 8	$\dfrac{\operatorname{erf}[ia^{\frac{1}{2}}(p+\lambda)^{\frac{1}{2}}]}{(p+\lambda)^{\frac{1}{2}}}$ E $a = \infty$ pair 527	$-\dfrac{1}{(\pi g)^{\frac{1}{2}}}e^{-\lambda g}$	$-a < g < 0$
908 3	$(p+\rho)^{\frac{1}{2}}\operatorname{erfc}[a^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}] - \dfrac{1}{(\pi a)^{\frac{1}{2}}}e^{-a(p+\rho)}$	$-\dfrac{1}{2\pi^{\frac{1}{2}}g^{\frac{3}{2}}}e^{-\rho g}$,	$a < g$
909 1 Key	$\exp(\sigma^2 p)K_\nu[\sigma^2(p+\rho)]$ S $\nu = 0\ \ \pm\frac{1}{2}\ \ \pm 1$ pairs 912 2 526 912 6 E $\sigma = \infty$ pair 526 R $\sigma = 0$ pair 524 2 C $(\nu+\frac{1}{2})$ an integer σ unrestricted by notation D $1 \leq \lvert R(\nu)\rvert$ $R(\rho) = 0$	$\dfrac{\exp[-\rho(g+\sigma^2)]}{g^{\frac{1}{2}}(g+2\sigma^2)^{\frac{1}{2}}} \times \cosh\left[\nu\cosh^{-1}\left(\dfrac{g+\sigma^2}{\sigma^2}\right)\right]$ $= \dfrac{\exp[-\rho(g+\sigma^2)]}{2g^{\frac{1}{2}}(g+2\sigma^2)^{\frac{1}{2}}} \times \left\{\left[\dfrac{g^{\frac{1}{2}}+(g+2\sigma^2)^{\frac{1}{2}}}{2^{\frac{1}{2}}\sigma}\right]^{2\nu} + \left[\dfrac{g^{\frac{1}{2}}+(g+2\sigma^2)^{\frac{1}{2}}}{2^{\frac{1}{2}}\sigma}\right]^{-2\nu}\right\}$	$0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
909.2	$\dfrac{\exp(\sigma^2 p)K_\nu[\sigma^2(p+\beta)]}{p+\beta}$ $\boxed{\text{S}}$ $\nu = 0, \pm\frac{1}{2}, \pm 1$: pairs 913.3, 529, 912.4 $\boxed{\text{S}}$ $\sigma = 0$: pair 524.2 with $1 \leqq R(\alpha)$ $\boxed{\text{R}}$ $\sigma = \infty$: pair 529 $\boxed{\text{C}}$ $(\nu + \frac{1}{2})$ an integer; σ unrestricted by notation	$\dfrac{1}{\nu}\exp[-\beta(g+\sigma^2)] \times \sinh\left[\nu \cosh^{-1}\left(\dfrac{g+\sigma^2}{\sigma^2}\right)\right]$ $= \dfrac{1}{2\nu}\exp[-\beta(g+\sigma^2)] \times \left\{\left[\dfrac{g^{1/2}+(g+2\sigma^2)^{1/2}}{2^{1/2}\sigma}\right]^{2\nu} - \left[\dfrac{g^{1/2}+(g+2\sigma^2)^{1/2}}{2^{1/2}\sigma}\right]^{-2\nu}\right\}, \quad 0 < g$
909.6	$K_\nu[x(p+\rho)]$ $\boxed{\text{S}}$ $\nu = 0, \pm 1$: pairs 912.3, 912.7 $\boxed{\text{R}}$ $x = 0$: pair 524.2 $\boxed{\text{D}}$ $1 \leqq \lvert R(\nu)\rvert$, $R(\rho) = 0$	$\dfrac{e^{-\rho g}}{(g^2-x^2)^{1/2}}\cosh\left[\nu\cosh^{-1}\left(\dfrac{g}{x}\right)\right]$ $= \dfrac{e^{-\rho g}}{2(g^2-x^2)^{1/2}}\left\{\left[\dfrac{(g-x)^{1/2}+(g+x)^{1/2}}{2^{1/2}x^{1/2}}\right]^{2\nu} + \left[\dfrac{(g-x)^{1/2}+(g+x)^{1/2}}{2^{1/2}x^{1/2}}\right]^{-2\nu}\right\}, \quad x < g$
909.7	$\dfrac{I_\nu[r(p+\rho)]}{p+\rho}$ $\boxed{\text{S}}$ $\nu = \pm 1$: pair 914.8 $\boxed{\text{R}}$ $\nu = 0$: pair 909.75 $\boxed{\text{R}}$ $r = 0$: pair 524.2 $\boxed{\text{C}}$ ν an integer; ρ unrestricted by notation $\boxed{\text{D}}$ $R(\nu) \leqq 0$, $R(\rho) = 0$, ν not a negative integer	$\begin{cases} \dfrac{1}{\pi\nu}e^{-\rho g}\sin\left[\nu\cos^{-1}\left(-\dfrac{g}{r}\right)\right] \\ = \dfrac{1}{i2\pi\nu}e^{-\rho g}\left\{\left[\dfrac{(r-g)^{1/2}+i(r+g)^{1/2}}{2^{1/2}r^{1/2}}\right]^{2\nu} - \left[\dfrac{(r-g)^{1/2}+i(r+g)^{1/2}}{2^{1/2}r^{1/2}}\right]^{-2\nu}\right\}, & \lvert g\rvert < r \\ \dfrac{\sin\pi\nu}{\pi\nu}e^{-\rho g}\exp\left[-\nu\cosh^{-1}\left(\dfrac{g}{r}\right)\right] \\ = \dfrac{\sin\pi\nu}{\pi\nu}e^{-\rho g}\left[\dfrac{(g-r)^{1/2}+(g+r)^{1/2}}{2^{1/2}r^{1/2}}\right]^{-2\nu}, & r < g \end{cases}$
909.75	$\dfrac{I_0[r(p+\beta)]}{p+\beta}$ $\boxed{\text{S}}$ $r = 0$: pair 438	$\begin{cases} \dfrac{1}{\pi}e^{-\beta g}\cos^{-1}\left(-\dfrac{g}{r}\right), & \lvert g\rvert < r \\ e^{-\beta g}, & r < g \end{cases}$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
909.8	$I_\nu[r(p+\rho)]$ $\nu = 0, \pm 1$, pairs 914.6, 909.9 $r = 0$, pair 524.2 ν an integer, ρ unrestricted by notation $R(\nu) \leqq -1$, $R(\rho) = 0$, ν not a negative integer	$\begin{cases} \dfrac{e^{-\rho g}}{\pi(r^2-g^2)^{\frac{1}{2}}}\cos\left[\nu\cos^{-1}\left(-\dfrac{g}{r}\right)\right] \\ = \dfrac{e^{-\rho g}}{2\pi(r^2-g^2)^{\frac{1}{2}}} \\ \quad\times\left\{\left[\dfrac{(r-g)^{\frac{1}{2}}+i(r+g)^{\frac{1}{2}}}{2^{\frac{1}{2}}r^{\frac{1}{2}}}\right]^{2\nu} + \left[\dfrac{(r-g)^{\frac{1}{2}}+i(r+g)^{\frac{1}{2}}}{2^{\frac{1}{2}}r^{\frac{1}{2}}}\right]^{-2\nu}\right\}, \quad \|g\| < r \\ -\dfrac{\sin\pi\nu\, e^{-\rho g}}{\pi(g^2-r^2)^{\frac{1}{2}}}\exp\left[-\nu\cosh^{-1}\left(\dfrac{g}{r}\right)\right] \\ = -\dfrac{\sin\pi\nu\, e^{-\rho g}}{\pi(g^2-r^2)^{\frac{1}{2}}} \\ \quad\times\left[\dfrac{(g-r)^{\frac{1}{2}}+(g+r)^{\frac{1}{2}}}{2^{\frac{1}{2}}r^{\frac{1}{2}}}\right]^{-2\nu}, \quad r < g \end{cases}$
909.9	$I_1[a(p+\lambda)]$	$-\dfrac{g e^{-\lambda g}}{\pi a(a^2-g^2)^{\frac{1}{2}}} \qquad \|g\| < a$
909.91	$e^{-ap}I_1[a(p+\lambda)]$ $a = \infty$, pair 526	$\dfrac{(a-g)e^{-\lambda(g-a)}}{\pi a g^{\frac{1}{2}}(2a-g)^{\frac{1}{2}}} \qquad 0 < g < 2a$
910.3	$\dfrac{K_\nu[x(p+\beta)]}{p+\beta}$ $\nu = 0, \pm 1$, pairs 913.1, 912.5 $x = 0$, pair 524.2 with $1 \leq R(\nu)$	$\dfrac{1}{\nu}e^{-\beta g}\sinh\left[\nu\cosh^{-1}\left(\dfrac{g}{x}\right)\right] = \dfrac{1}{2\nu}e^{-\beta g}\left\{\left[\dfrac{(g-x)^{\frac{1}{2}}+(g+x)^{\frac{1}{2}}}{(2x)^{\frac{1}{2}}}\right]^{2\nu} - \left[\dfrac{(g-x)^{\frac{1}{2}}+(g+x)^{\frac{1}{2}}}{(2x)^{\frac{1}{2}}}\right]^{-2\nu}\right\} \quad x < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
911.1	$\frac{1}{(p+\rho)^{\alpha-\frac{1}{2}}}\exp(\sigma^2 p)K_{\alpha-\frac{1}{2}}[\sigma^2(p+\rho)]$ [S] $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$: pairs 911.3, 912.2, 911.4, 438 [B] $\sigma = \infty$: pair 524.2 [R] $\sigma = 0$: pair 524.2 [C] α an integer; σ unrestricted by notation [D] $1 \leqq R(\alpha)$, $R(\rho) = 0$	$\frac{\pi^{\frac{1}{2}}\exp[-\rho(g+\sigma^2)]}{2^{\alpha-\frac{1}{2}}\sigma^{2\alpha-1}\Gamma(\alpha)}g^{\alpha-1}(g+2\sigma^2)^{\alpha-1}$, $0 < g$
911.3	$(p+\rho)^{\frac{1}{4}}\exp(\sigma^2 p)K_{\frac{1}{4}}[\sigma^2(p+\rho)]$ [B] $\sigma = \infty$: pair 526.4 [D] $\sigma = 0$	$\frac{2^{\frac{1}{4}}\pi^{\frac{1}{2}}\sigma^{\frac{1}{2}}\exp[-\rho(g+\sigma^2)]}{\Gamma(\frac{1}{4})g^{\frac{3}{4}}(g+2\sigma^2)^{\frac{3}{4}}}$, $0 < g$
911.4	$\frac{1}{(p+\rho)^{\frac{1}{4}}}\exp(\sigma^2 p)K_{\frac{1}{4}}[\sigma^2(p+\rho)]$ [S] $\sigma = 0$: pair 526 [B] $\sigma = \infty$: pair 526.7	$\frac{\pi^{\frac{1}{2}}\exp[-\rho(g+\sigma^2)]}{2^{\frac{1}{4}}\Gamma(\frac{3}{4})\sigma^{\frac{1}{2}}g^{\frac{1}{4}}(g+2\sigma^2)^{\frac{1}{4}}}$, $0 < g$
911.6	$\frac{K_{\alpha-\frac{1}{2}}[x(p+\rho)]}{(p+\rho)^{\alpha-\frac{1}{2}}}$ [S] $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}$: pairs 911.7, 912.3, 911 8, 604, 912.5 [R] $x = 0$: pair 524.2 [D] $1 \leqq R(\alpha)$, $R(\rho) = 0$	$\frac{\pi^{\frac{1}{2}}}{(2x)^{\alpha-\frac{1}{2}}\Gamma(\alpha)}e^{-\rho g}(g^2-x^2)^{\alpha-1}$, $x < g$
911.7	$(p+\rho)^{\frac{1}{4}}K_{\frac{1}{4}}[x(p+\rho)]$ [D] $x = 0$	$\frac{2^{\frac{1}{4}}\pi^{\frac{1}{2}}x^{\frac{1}{4}}e^{-\rho g}}{\Gamma(\frac{1}{4})(g^2-x^2)^{\frac{3}{4}}}$, $x < g$
911.8	$\frac{K_{\frac{1}{4}}[x(p+\rho)]}{(p+\rho)^{\frac{1}{4}}}$ [S] $x = 0$: pair 526	$\frac{\pi^{\frac{1}{2}}e^{-\rho g}}{2^{\frac{1}{4}}\Gamma(\frac{3}{4})x^{\frac{1}{4}}(g^2-x^2)^{\frac{1}{4}}}$, $x < g$

No	Coefficient $F(f)$	Coefficient $G(g)$	
912.2	$\exp(\sigma^2 p)K_0[\sigma^2(p+\rho)]$ $\sigma = \infty$ pair 526 $\sigma = 0$	$\dfrac{\exp[-\rho(g+\sigma^2)]}{g^{\frac{1}{2}}(g+2\sigma^2)^{\frac{1}{2}}}$,	$0 < g$
912.3	$K_0[x(p+\rho)]$ $x = 0$	$\dfrac{e^{-\rho g}}{(g^2-x^2)^{\frac{1}{2}}}$,	$x < g$
912.4	$\dfrac{\exp(\sigma^2 p)K_1[\sigma^2(p+\beta)]}{p+\beta}$ $\sigma = 0$ pair 442 $\sigma = \infty$ pair 529	$\dfrac{1}{\sigma^2}g^{\frac{1}{2}}(g+2\sigma^2)^{\frac{1}{2}}\exp[-\beta(g+\sigma^2)]$	$0 < g$
912.5	$\dfrac{K_1[x(p+\beta)]}{p+\beta}$ $x = 0$ pair 442	$\dfrac{1}{x}e^{-\beta g}(g^2-x^2)^{\frac{1}{2}}$,	$x < g$
912.6	$\exp(\sigma^2 p)K_1[\sigma^2(p+\beta)]$ $\sigma = 0$ pair 438 $\sigma = \infty$ pair 526	$\dfrac{(g+\sigma^2)\exp[-\beta(g+\sigma^2)]}{\sigma^2 g^{\frac{1}{2}}(g+2\sigma^2)^{\frac{1}{2}}}$	$0 < g$
912.7	$K_1[x(p+\beta)]$ $x = 0$ pair 438	$\dfrac{g e^{-\beta g}}{x(g^2-x^2)^{\frac{1}{2}}}$	$x < g$
913	$\dfrac{1}{p}K_0(xp) = \lim_{\beta\to 0}\left\{\dfrac{K_0[x(p+\beta)]}{p+\beta}\right\}$	$\cosh^{-1}\left(\dfrac{g}{x}\right)$ $= 2\log\left[\dfrac{(g-x)^{\frac{1}{2}}+(g+x)^{\frac{1}{2}}}{(2x)^{\frac{1}{2}}}\right]$	$x < g$
913.1	$\dfrac{K_0[x(p+\beta)]}{p+\beta}$ $x = 0$ pair 438 $\beta \to 0$ pair 913	$e^{-\beta g}\cosh^{-1}\left(\dfrac{g}{x}\right)$ $= 2e^{-\beta g}\log\left[\dfrac{(g-x)^{\frac{1}{2}}+(g+x)^{\frac{1}{2}}}{(2x)^{\frac{1}{2}}}\right]$	$x < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$	
913.3	$\dfrac{\exp(\sigma^2 p)K_0[\sigma^2(p+\beta)]}{p+\beta}$ $\boxed{S}$ $\sigma = 0$: pair 438 $\boxed{B}$ $\sigma = \infty$: pair 529	$\exp[-\beta(g+\sigma^2)]\cosh^{-1}\left(\dfrac{g+\sigma^2}{\sigma^2}\right)$ $= 2\exp[-\beta(g+\sigma^2)] \times \log\left[\dfrac{g^{\frac{1}{2}}+(g+2\sigma^2)^{\frac{1}{2}}}{2^{\frac{1}{2}}\sigma}\right]$,	$0 < g$
914.2	$\dfrac{I_{\alpha-\frac{1}{2}}[c(p+\lambda)]}{(p+\lambda)^{\alpha-\frac{1}{2}}}$ $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}$: pairs 914.4, 914.6, 914.9, 622.1, 914.8	$\dfrac{e^{-\lambda g}(c^2-g^2)^{\alpha-1}}{\pi^{\frac{1}{2}}(2c)^{\alpha-\frac{1}{2}}\Gamma(\alpha)}$,	$\lvert g\rvert < c$
914.3	$\dfrac{I_n(xp)}{p^n}$ $\boxed{S}$ $n = 0, 1$: pairs 914.5, 914.7 $\boxed{O}$ $x = 0$	$\dfrac{(x^2-g^2)^{n-\frac{1}{2}}}{\pi^{\frac{1}{2}}\Gamma(n+\frac{1}{2})(2x)^n}$,	$\lvert g\rvert < \lvert x\rvert$
914.4	$(p+\lambda)^{\frac{1}{4}}I_{-\frac{1}{4}}[a(p+\lambda)]$	$\dfrac{(2a)^{\frac{1}{4}}e^{-\lambda g}}{\Gamma(\frac{1}{4})\pi^{\frac{1}{2}}(a^2-g^2)^{\frac{3}{4}}}$,	$\lvert g\rvert < a$
914.5	$I_0(xp)$ $\boxed{O}$ $x = 0$	$\dfrac{1}{\pi(x^2-g^2)^{\frac{1}{2}}}$,	$\lvert g\rvert < \lvert x\rvert$
914.6	$I_0[a(p+\lambda)]$ $\boxed{S}$ $\lambda = 0$: pair 914.5	$\dfrac{e^{-\lambda g}}{\pi(a^2-g^2)^{\frac{1}{2}}}$,	$\lvert g\rvert < a$
914.61	$e^{-ap}I_0[a(p+\lambda)]$ $\boxed{B}$ $a = \infty$: pair 526	$\dfrac{e^{-\lambda(g-a)}}{\pi g^{\frac{1}{2}}(2a-g)^{\frac{1}{2}}}$,	$0 < g < 2a$
914.7	$\dfrac{I_1(xp)}{p}$ $\boxed{O}$ $x = 0$	$\dfrac{1}{\pi x}(x^2-g^2)^{\frac{1}{2}}$,	$\lvert g\rvert < \lvert x\rvert$

TABLE I (Continued)

No	Coefficient $F(f)$		Coefficient $G(g)$	
914 8	$\frac{I_1[a(p+\lambda)]}{p+\lambda}$		$\frac{1}{\pi a}e^{-\lambda g}(a^2-g^2)^{\frac{1}{2}}$,	$\|g\| < a$
	⊠ $\lambda = 0$ pair 914 7			
914 9	$\frac{I_{\frac{1}{2}}[a(p+\lambda)]}{(p+\lambda)^{\frac{1}{2}}}$		$\frac{e^{-\lambda g}}{\Gamma(\frac{1}{2})\pi^{\frac{1}{2}}(2a)^{\frac{1}{2}}(a^2-g^2)^{\frac{1}{2}}}$,	$\|g\| < a$
915 4	$J_0[x(\rho^2-p^2)^{\frac{1}{2}}]$		$\frac{\cos[\rho(x^2-g^2)^{\frac{1}{2}}]}{\pi(x^2-g^2)^{\frac{1}{2}}}$,	$\|g\| < \|x\|$
	⊠ $\rho = 0$ pair 914 5 ⊠ $x = \infty$ and $\rho = \infty$ pair 710 0 ⊠ $x = 0$			
915 7	$\frac{J_1[x(\rho^2-p^2)^{\frac{1}{2}}]}{(\rho^2-p^2)^{\frac{1}{2}}}$		$\frac{1}{\pi x\rho}\sin[\rho(x^2-g^2)^{\frac{1}{2}}]$,	$\|g\| < \|x\|$
	⊠ $\rho = 0$ pair 914 7 ⊠ $x = \infty$ and $\rho = \infty$ pair 710 0 ⊠ $x = 0$			
916	$\frac{K_1[2\pi\rho(f^2+\rho^2)^{\frac{1}{2}}]}{(f^2+\rho^2)^{\frac{1}{2}}}$		$\frac{K_1[2\pi\rho(g^2+\rho^2)^{\frac{1}{2}}]}{(g^2+\rho^2)^{\frac{1}{2}}}$	
	⊠ $\rho = 0$ pair 523 ⊠ $\rho = \infty$ pair 705 1			
916.3	$\frac{\|f\|J_{-1}[2\pi r(f^2-r^2)^{\frac{1}{2}}]}{(f^2-r^2)^{\frac{1}{2}}}$,	$r < \|f\|$	$\frac{\|g\|J_{-1}[2\pi r(g^2-r^2)^{\frac{1}{2}}]}{(g^2-r^2)^{\frac{1}{2}}}$,	$r < \|g\|$
	⊠ $r = 0$ pair 523			
916.5	$\frac{K_1[\sigma(\rho^2-p^2)^{\frac{1}{2}}]}{(\rho^2-p^2)^{\frac{1}{2}}}$		$\frac{\rho^{\frac{1}{2}}K_{\frac{1}{2}}[\rho(g^2+\sigma^2)^{\frac{1}{2}}]}{(2\pi)^{\frac{1}{2}}\sigma^{\frac{1}{2}}(g^2+\sigma^2)^{\frac{1}{2}}}$	
	⊠ $\rho = 0$ pair 918 8 ⊠ $\sigma = 0$ pair 558 8 ⊠ $\rho = 2\pi\sigma$ pair 916 ⊠ $\rho = \infty$ and $\sigma = \infty$ pair 710 0			

No.	Coefficient $F(f)$	Coefficient $G(g)$
916.7	$\dfrac{\lvert f\rvert J_{-\alpha-\frac12}[2\pi s(f^2-r^2)^{\frac12}]}{(f^2-r^2)^{\frac12\alpha+\frac14}}, \quad r<\lvert f\rvert$ $\boxed{S}$ $r=0$: pair 916.8 $\boxed{S}$ $r=s=0$: pair 522.5 $\boxed{S}$ $r=s,\ \alpha=\frac14$: pair 916.3 $\boxed{C}$ $\frac12 \leqq R(\alpha)$	$\dfrac{r^{\frac12-\alpha}\lvert g\rvert J_{\alpha-\frac12}[2\pi r(g^2-s^2)^{\frac12}]}{s^{\alpha+\frac12}(g^2-s^2)^{\frac14-\frac12\alpha}}, \quad s<\lvert g\rvert$
916.8	$\lvert f\rvert^{\frac12-\alpha}J_{-\alpha-\frac12}(2\pi s\lvert f\rvert)$ $\boxed{S}$ $s=0$: pair 522.5 $\boxed{C}$ $\frac12 \leqq R(\alpha)$	$\dfrac{\lvert g\rvert(g^2-s^2)^{\alpha-\frac12}}{\pi^{\frac12-\alpha}s^{\alpha+\frac12}\Gamma(\alpha)}, \quad s<\lvert g\rvert$
917	$K_0[\sigma(\rho^2-p^2)^{\frac12}]$ $\boxed{S}$ $\rho=0$: pair 918 $\boxed{B}$ $\rho=\infty$ and $\sigma=\infty$: pair 710.0 $\boxed{C}$ $\sigma=0$	$\dfrac{\exp[-\rho(g^2+\sigma^2)^{\frac12}]}{2(g^2+\sigma^2)^{\frac12}}$
917.5	$\dfrac{K_1[\sigma(\beta^2-p^2)^{\frac12}]}{(\beta^2-p^2)^{\frac12}}$ $\boxed{S}$ $\sigma=0$: pair 444 $\boxed{B}$ $\beta=\infty$ and $\sigma=\infty$: pair 710.0	$\dfrac{1}{2\beta\sigma}\exp[-\beta(g^2+\sigma^2)^{\frac12}]$
917.8	$(\rho^2-p^2)^{\frac12}K_1[\alpha(\rho^2-p^2)^{\frac12}]$ $\boxed{S}$ $\rho=0$: pair 919 $\boxed{B}$ $\alpha=\infty$ and $\rho=\infty$: pair 710.0	$\dfrac{\alpha[1+\rho(g^2+\alpha^2)^{\frac12}]}{2(g^2+\alpha^2)^{\frac32}}\exp[-\rho(g^2+\alpha^2)^{\frac12}]$
918	$K_0(\sigma\lvert p\rvert)$ $\boxed{C}$ $\sigma=0$	$\dfrac{1}{2(g^2+\sigma^2)^{\frac12}}$
918.5	$\lvert p\rvert^{\alpha-\frac12}K_{\alpha-\frac12}(\sigma\lvert p\rvert)$ $\boxed{S}$ $\alpha=\frac14,\frac12,1,\frac32$: pairs 918.8, 918, 632, 919 $\boxed{B}$ $\sigma=0$: pair 522.5 $\boxed{C}$ $1 \leqq R(\alpha)$, $R(\sigma)=0$	$\dfrac{2^{\alpha-\frac12}\sigma^{\alpha-\frac12}\Gamma(\alpha)}{\pi^{\frac12}(g^2+\sigma^2)^{\alpha}}$

TABLE 1 (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$	
918.8	$\dfrac{K_{\frac14}(\sigma\lvert p\rvert)}{\lvert p\rvert^{\frac14}}$	$\dfrac{\Gamma(\frac34)}{2\pi^{\frac12}\sigma^{\frac14}(g^2+\sigma^2)^{\frac34}}$	
	$\sigma = 0$ pair 523.1		
919	$\lvert p\rvert K_1(\alpha\lvert p\rvert)$	$\dfrac{\alpha}{2(g^2+\alpha^2)^{\frac32}}$	
919.5	$\lvert p\rvert^{\alpha-\frac12}J_{\alpha-\frac12}(x\lvert p\rvert)$	$\dfrac{\Gamma(\alpha)(2x)^{\alpha-\frac12}}{\pi^{\frac12}(x^2-g^2)^{\alpha}}$	$\lvert g\rvert < \lvert x\rvert$
	$\alpha = \frac12$ pair 914.3 $\alpha = -\frac12$ pair 914.7 $\alpha = \frac12 - n$ pair 914.3 $x = 0$ pair 522.5 $1 \leq R(\alpha)$	$\dfrac{\cos(\pi\alpha)\Gamma(\alpha)(2x)^{\alpha-\frac12}}{\pi^{\frac12}(g^2-x^2)^{\alpha}},$	$\lvert x\rvert < \lvert g\rvert$
920.1	$K_0[\sigma^{\frac12}(p+\rho)^{\frac12}]$	$\dfrac{1}{2g}\exp\left(-\rho g-\dfrac{\sigma}{4g}\right)$	$0 < g$
	$\sigma = 0$		
921.1	$(p+\rho)^{\frac12}K_1[\alpha^{\frac12}(p+\rho)^{\frac12}]$	$\dfrac{\alpha^{\frac12}}{4g^2}\exp\left(-\rho g-\dfrac{\alpha}{4g}\right)$	$0 < g$
922.1	$\dfrac{K_1[\sigma^{\frac12}(p+\rho)^{\frac12}]}{(p+\rho)^{\frac12}}-\dfrac{1}{\sigma^{\frac12}(p+\rho)}$	$\dfrac{1}{\sigma^{\frac12}}e^{-\rho g}\left[\exp\left(-\dfrac{\sigma}{4g}\right)-1\right],$	$0 < g$
	$\sigma = \infty$ pair 438 $\sigma = 0$		
922.2	$\dfrac{K_1[\sigma^{\frac12}(p+\beta)^{\frac12}]}{(p+\beta)^{\frac12}}$	$\dfrac{1}{\sigma^{\frac12}}\exp\left(-\beta g-\dfrac{\sigma}{4g}\right)$	$0 < g$
	$\sigma = 0$ pair 438		
923.1	$(p+\lambda)^{\frac12}\exp[\sigma(p+\lambda)^2]K_{\frac14}[\sigma(p+\lambda)^2]$	$\dfrac{1}{(2\sigma g)^{\frac12}}\exp\left(-\lambda g-\dfrac{g^2}{8\sigma}\right)$	$0 < g$
	$\sigma = \infty$ pair 526 $\sigma = 0$ $R(\lambda) < 0$ $R(\sigma) = 0$		

No.	Coefficient $F(f)$	Coefficient $G(g)$
923.2	$\dfrac{1}{(p+\beta)^{\frac{1}{2}}}\exp[\sigma(p+\beta)^2]K_{\frac{1}{4}}[\sigma(p+\beta)^2]$ $\boxed{\text{S}}$ $\sigma = 0$: pair 438 $\boxed{\text{B}}$ $\sigma = \infty$: pair 529	$\dfrac{1}{2^{\frac{1}{4}}\sigma^{\frac{1}{4}}}e^{-\beta g}\gamma\left(\dfrac{1}{4}, \dfrac{g^2}{8\sigma}\right)$, $\quad 0 < g$
923.4	$\exp[\sigma(p+\rho)^2]K_0[\sigma(p+\rho)^2]$ $\boxed{\text{B}}$ $\sigma = \infty$: pair 438 $\boxed{\text{D}}$ $\sigma = 0$	$\left(\dfrac{\pi}{2\sigma}\right)\exp\left(-\rho g - \dfrac{g^2}{16\sigma}\right)I_0\left(\dfrac{g^2}{16\sigma}\right)$, $0 < g$
923.5	$\exp(-\sigma^3 p^2)K_0(\sigma^3\|p\|^2)$ $\boxed{\text{S}}$ $\sigma = 1/(2\pi^{\frac{1}{4}})$: pair 923.55 $\boxed{\text{D}}$ $\sigma = 0$	$\dfrac{1}{\pi^{\frac{1}{2}}(2\sigma)^{\frac{1}{2}}}\exp\left(\dfrac{g^2}{16\sigma^3}\right)K_0\left(\dfrac{\|g\|^2}{16\sigma^3}\right)$
923.55	$\exp(\frac{1}{2}\pi f^2)K_0(\frac{1}{2}\pi\|f\|^2)$	$\exp(\frac{1}{2}\pi g^2)K_0(\frac{1}{2}\pi\|g\|^2)$
923.6	$\dfrac{1}{(p+\rho)^{2\alpha-\frac{1}{2}}}\exp[\sigma(p+\rho)^2]$ $\times K_{\alpha-\frac{1}{2}}[\sigma(p+\rho)^2]$ $\boxed{\text{S}}$ $\alpha = \frac{1}{2}, \frac{3}{4}, 1$: pairs 923.4, 923.2, 442 $\boxed{\text{B}}$ $\alpha = \frac{1}{4}$: pair 923.1 $\boxed{\text{B}}$ $\sigma = \infty$: pair 524.2 $\boxed{\text{R}}$ $\sigma = 0$: pair 524.2 $\boxed{\text{C}}$ α an integer; σ unrestricted by notation $\boxed{\text{D}}$ $\frac{3}{4} \leqq R(\alpha)$, $R(\rho) = 0$	$\dfrac{2^{\frac{3}{2}\alpha+\frac{1}{4}}\pi^{\frac{1}{2}}\sigma^{\frac{1}{2}\alpha-\frac{1}{4}}}{\Gamma(2\alpha)}g^{\alpha-\frac{3}{2}}\exp\left(-\rho g - \dfrac{g^2}{16\sigma}\right)$ $\times M_{\frac{3}{4}-\frac{3}{2}\alpha,\,\frac{1}{2}\alpha-\frac{1}{4}}\left(\dfrac{g^2}{8\sigma}\right)$, $\quad 0 < g$
923.8 Key	$p^{\frac{1}{2}}J_{-\frac{1}{4}}(ap^2)$ $\boxed{\text{S}}$ $a = 1/(4\pi)$: pair 923.85	$\dfrac{g^{\frac{1}{2}}}{2^{\frac{3}{2}}\pi^{\frac{1}{2}}a}J_{-\frac{1}{4}}\left(\dfrac{g^2}{4a}\right)$
923.85	$f^{\frac{1}{2}}J_{-\frac{1}{4}}(\pi f^2)$	$g^{\frac{1}{2}}J_{-\frac{1}{4}}(\pi g^2)$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
924.1	$\frac{1}{(p+\rho)^{\frac12}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times K_{\alpha-\frac12}\left[\frac{1}{\lambda(p+\rho)}\right]$ $\alpha = \frac12$ pair 924.5 $\rho = 0$ pair 924.2 $\lambda = \infty$ pair 524.2 $\alpha = \frac14$ or $\alpha = \frac34$ pair 924.3 $1 \leqq R(\alpha)$ λ 0 $\pi < \lvert\arg\lambda\rvert$ $R(\rho) = 0$	$\frac{2\sin\pi\alpha}{(\pi g)^{\frac12}}e^{-\rho g}K_{2\alpha-1}\left(\frac{2^{\frac32}g^{\frac12}}{\lambda^{\frac12}}\right)$	$0 < g$
924.2	$\frac{1}{p^{\frac12}}\exp\left(\frac{1}{\sigma^2 p}\right)K_{\alpha-\frac12}\left(\frac{1}{\sigma^2 p}\right)$ $\alpha = \frac14$ or $\frac34$, $\frac12$ pairs 924.4, 924.6 $\sigma = \infty$ pair 521 $1 \leqq R(\alpha)$ $\sigma = 0$	$\frac{2\sin\pi\alpha}{(\pi g)^{\frac12}}K_{2\alpha-1}\left(\frac{2^{\frac32}g^{\frac12}}{\sigma}\right)$	$0 < g$
924.3	$\frac{1}{(p+\rho)^{\frac12}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times K_{\frac14}\left[\frac{1}{\lambda(p+\rho)}\right]$ $\rho = 0$ pair 924.4 $\lambda = \infty$ pair 526.4 $\lambda = 0$ $R(\lambda^{\frac12}) < 0$ $R(\rho) = 0$	$\frac{\lambda^{\frac14}}{(2g)^{\frac34}}\exp\left(-\rho g - \frac{2^{\frac32}g^{\frac12}}{\lambda^{\frac12}}\right)$	$0 < g$
924.4	$\frac{1}{p^{\frac12}}\exp\left(\frac{1}{\sigma^2 p}\right)K_{\frac14}\left(\frac{1}{\sigma^2 p}\right)$ $\sigma = \infty$ pair 521.4 $\sigma = 0$	$\frac{\sigma^{\frac12}}{(2g)^{\frac34}}\exp\left(-\frac{2^{\frac32}g^{\frac12}}{\sigma}\right)$	$0 < g$
924.5	$\frac{1}{(p+\rho)^{\frac12}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times K_0\left[\frac{1}{\lambda(p+\rho)}\right]$ $\rho = 0$ pair 924.6 $\lambda = \infty$ pair 526 $\lambda = 0$ $\pi < \lvert\arg\lambda\rvert$ $R(\rho) = 0$	$\frac{2}{(\pi g)^{\frac12}}e^{-\rho g}K_0\left(\frac{2^{\frac32}g^{\frac12}}{\lambda^{\frac12}}\right)$	$0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
924.6	$\frac{1}{p^{\frac{1}{2}}}\exp\left(\frac{1}{\sigma^2 p}\right)K_0\left(\frac{1}{\sigma^2 p}\right)$ $\boxed{B}$ $\sigma = \infty$: pair 522 $\boxed{D}$ $\sigma = 0$	$\frac{2}{(\pi g)^{\frac{1}{2}}}K_0\left(\frac{2^{\frac{3}{2}}g^{\frac{1}{2}}}{\sigma}\right)$,	$0 < g$
925.1	$\frac{1}{p^{\frac{1}{2}}}\exp\left(-\frac{1}{cp}\right)I_{\alpha-\frac{1}{2}}\left(\frac{1}{cp}\right)$ $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{3}{4}, 1$: pairs 926.7, 926.4, 654.3 $\boxed{B}$ $c = \infty$: pair 521	$\frac{1}{(\pi g)^{\frac{1}{2}}}J_{2\alpha-1}\left(\frac{2^{\frac{3}{2}}g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$0 < g$
926.0	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times I_{\alpha-\frac{1}{2}}\left[\frac{1}{\lambda(p+\rho)}\right]$ $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{3}{4}, 1$: pairs 926.6, 926.3, 654.2 $\boxed{S}$ $\rho = 0$: pair 925.1 $\boxed{B}$ $\lambda = \infty$: pair 524.2 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$	$\frac{1}{(\pi g)^{\frac{1}{2}}}e^{-\rho g}I_{2\alpha-1}\left(\frac{2^{\frac{3}{2}}g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
926.3	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times I_{\frac{1}{4}}\left[\frac{1}{\lambda(p+\rho)}\right]$ $\boxed{S}$ $\rho = 0$: pair 926.4 $\boxed{B}$ $\lambda = \infty$: pair 526.7 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$	$\frac{\lambda^{\frac{1}{4}}}{2^{\frac{1}{4}}\pi g^{\frac{3}{4}}}e^{-\rho g}\sinh\left(\frac{2^{\frac{3}{2}}g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
926.4	$\frac{1}{p^{\frac{1}{2}}}\exp\left(-\frac{1}{cp}\right)I_{\frac{1}{4}}\left(\frac{1}{cp}\right)$ $\boxed{B}$ $c = \infty$: pair 521.7	$\frac{c^{\frac{1}{4}}}{2^{\frac{1}{4}}\pi g^{\frac{3}{4}}}\sin\left(\frac{2^{\frac{3}{2}}g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$,	$0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
926.6	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times I_{-\frac{1}{2}}\left[\frac{1}{\lambda(p+\rho)}\right]$ $\rho = 0$ pair 926.7 $\lambda = \infty$ pair 526.4 $\lambda = 0$ $\lambda \neq -\lvert\lambda\rvert,\ R(\rho) = 0$	$\frac{\lambda^{\frac{1}{2}}}{2^{\frac{1}{2}}\pi g^{\frac{1}{2}}} e^{-\rho g}\cosh\left(\frac{2^{\frac{3}{2}} g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$, $\quad 0 < g$
926.7	$\frac{1}{p^{\frac{1}{2}}}\exp\left(-\frac{1}{cp}\right) I_{-\frac{1}{2}}\left(\frac{1}{cp}\right)$ $c = \infty$ pair 521.4	$\frac{c^{\frac{1}{2}}}{2^{\frac{1}{2}}\pi g^{\frac{1}{2}}}\cos\left(\frac{2^{\frac{3}{2}} g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$, $\quad 0 < g$
927.0	$(p+\rho)^{\frac{1}{2}\nu} K_\nu[\sigma^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}]$ $\nu = -1,\ -\frac{1}{2},\ 0,\ \frac{1}{2},\ 1$ pairs 922.2, 823, 920.1, 817, 921.1 $\sigma = 0$ pair 524.2 $R(\nu) \leqq -1,\ R(\sigma) = 0$ $\frac{1}{2} \leqq R(\nu)$ [$1 \leqq R(\nu)$ for transposed pair] $R(\sigma) = 0$	$\frac{\sigma^{\frac{1}{2}\nu}}{(2g)^{\nu+1}}\exp\left(-\rho g - \frac{\sigma}{4g}\right)$, $\quad 0 < g$
928.2	$\frac{1}{(p+\rho)}\exp\left(\frac{\lambda+\mu}{p+\rho}\right) I_{n-1}\left(\frac{\lambda-\mu}{p+\rho}\right)$ $\lambda = 0$ or $\mu = 0$ pair 928.4 $\rho = 0$ pair 928.3 $\lambda = \mu$ pair 650.0 $\lambda \neq -\lvert\lambda\rvert$ or $\mu \neq -\lvert\mu\rvert,\ R(\rho) = 0$	$e^{-\rho g} I_{n-1}[2^{\frac{1}{2}}(\lambda^{\frac{1}{2}}+\mu^{\frac{1}{2}})g^{\frac{1}{2}}] \times I_{n-1}[2^{\frac{1}{2}}(\lambda^{\frac{1}{2}}-\mu^{\frac{1}{2}})g^{\frac{1}{2}}] \quad 0 < g$
928.3	$\frac{1}{p}\exp\left(-\frac{r+s}{p}\right) I_{n-1}\left(\frac{r-s}{p}\right)$ $r = 0$ or $s = 0$ pair 928.5 $r = s$ pair 650.4	$J_{n-1}[2^{\frac{1}{2}}(r^{\frac{1}{2}}+s^{\frac{1}{2}})g^{\frac{1}{2}}] J_{n-1}[2^{\frac{1}{2}}(r^{\frac{1}{2}}-s^{\frac{1}{2}})g^{\frac{1}{2}}]$, $\quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
928.4	$\frac{1}{p+\rho}\exp\left[\frac{1}{\lambda(p+\rho)}\right] \times I_{\alpha-1}\left[\frac{1}{\lambda(p+\rho)}\right]$ Ⓢ $\rho = 0$: pair 928.5 Ⓑ $\lambda = \infty$: pair 524.2 Ⓓ $\lambda = 0$ Ⓓ $\lambda \neq -\|\lambda\|$, $R(\rho) = 0$	$e^{-\rho g}\left[I_{\alpha-1}\left(\frac{2^{\frac{1}{2}}g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)\right]^2$, $\quad 0 < g$
928.5	$\frac{1}{p}\exp\left(-\frac{1}{cp}\right)I_{\alpha-1}\left(\frac{1}{cp}\right)$ Ⓑ $c = \infty$: pair 521	$\left[J_{\alpha-1}\left(\frac{2^{\frac{1}{2}}g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)\right]^2$, $\quad 0 < g$
929.1 Key	$\frac{\exp(\tau^2 p)K_{\alpha-\frac{1}{2}}[\tau^2(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}\alpha-\frac{1}{4}}(p+\sigma)^{\frac{1}{2}\alpha-\frac{1}{4}}}$ Ⓢ $\alpha = 1$: pair 860.5 Ⓑ $\rho = \infty$: or $\sigma = \infty$: pair 927.0 Ⓑ $\tau = \infty$: pair 650.0 Ⓡ $\rho = \sigma$: pair 911.1 Ⓡ $\tau = 0$: pair 570.1 Ⓒ α an integer; τ unrestricted by notation Ⓓ $\frac{3}{2} \leqq R(\alpha)$, $R(\rho) = 0$ or $R(\sigma) = 0$	$\frac{2^{\alpha-\frac{3}{2}}\pi^{\frac{1}{2}}}{\tau^{2\alpha-1}(\rho-\sigma)^{\alpha-1}}g^{\frac{1}{2}\alpha-\frac{3}{4}}(g+2\tau^2)^{\frac{1}{2}\alpha-\frac{3}{4}}$ $\times \exp[-\frac{1}{2}(\rho+\sigma)(g+\tau^2)]$ $\times I_{\alpha-1}[\frac{1}{2}(\rho-\sigma)g^{\frac{1}{2}}(g+2\tau^2)^{\frac{1}{2}}]$ $0 < g$
929.4	$\frac{K_{\alpha-\frac{1}{2}}[x(p+\rho)^{\frac{1}{2}}(p+\sigma)^{\frac{1}{2}}]}{(p+\rho)^{\frac{1}{2}\alpha-\frac{1}{4}}(p+\sigma)^{\frac{1}{2}\alpha-\frac{1}{4}}}$ Ⓢ $\alpha = 1$: pair 860.0 Ⓑ $\rho = \infty$ or $\sigma = \infty$: pair 927.0 Ⓡ $x = 0$: pair 570.1 Ⓡ $\rho = \sigma$: pair 911.6 Ⓓ $\frac{3}{2} \leqq R(\alpha)$, $R(\rho) = 0$ or $R(\sigma) = 0$	$\frac{2^{\alpha-\frac{3}{2}}\pi^{\frac{1}{2}}}{x^{\alpha-\frac{1}{2}}(\rho-\sigma)^{\alpha-1}}e^{-\frac{1}{2}(\rho+\sigma)g}(g^2-x^2)^{\frac{1}{2}\alpha-\frac{3}{4}}$ $\times I_{\alpha-1}[\frac{1}{2}(\rho-\sigma)(g^2-x^2)^{\frac{1}{2}}]$, $\quad x < g$
930.4	$\frac{I_{\alpha-\frac{1}{2}}[c(p+\lambda)^{\frac{1}{2}}(p+\mu)^{\frac{1}{2}}]}{(p+\lambda)^{\frac{1}{2}\alpha-\frac{1}{4}}(p+\mu)^{\frac{1}{2}\alpha-\frac{1}{4}}}$, Ⓢ $\alpha = 1$: pair 872.2 Ⓢ $\lambda = \mu$: pair 914.2 Ⓢ $\lambda = -\mu$: pair 930.5 or pair 930.7	$\frac{2^{\alpha-\frac{3}{2}}e^{-\frac{1}{2}(\lambda+\mu)g}}{\pi^{\frac{1}{2}}c^{\alpha-\frac{1}{2}}(\lambda-\mu)^{\alpha-1}}(c^2-g^2)^{\frac{1}{2}\alpha-\frac{3}{4}}$ $\times J_{\alpha-1}[\frac{1}{2}(\lambda-\mu)(c^2-g^2)^{\frac{1}{2}}]$, $\quad \|g\| < c$

No	Coefficient $F(f)$	Coefficient $G(g)$
930.5	$\dfrac{I_{\alpha-\frac12}[c(p^2+\lambda^2)^{\frac12}]}{(p^2+\lambda^2)^{\frac12\alpha-\frac14}}$ pair 930.7 $\alpha = 1$ pair 872.1	$\dfrac{(c^2-g^2)^{\frac12\alpha-\frac12}I_{\alpha-1}[\lambda(c^2-g^2)^{\frac12}]}{(2\pi)^{\frac12}c^{\alpha-\frac12}\lambda^{\alpha-1}}$, $\lvert g\rvert < c$
930.6	$\dfrac{J_n[x(\rho^2-p^2)^{\frac12}]}{(\rho^2-p^2)^{\frac12 n}}$ $n = 0, 1$ pairs 915.4, 915.7 $\rho = 0$ pair 914.3 $x = \infty$ and $\rho = \infty$ pair 710.0 $x = 0$	$\dfrac{(x^2-g^2)^{\frac12 n-\frac14}J_{n-\frac12}[\rho(x^2-g^2)^{\frac12}]}{(2\pi)^{\frac12}x^n\rho^{n-\frac12}}$, $\lvert g\rvert < \lvert x\rvert$
930.7	$\dfrac{J_{\alpha-\frac12}[c(\lambda^2-p^2)^{\frac12}]}{(\lambda^2-p^2)^{\frac12\alpha-\frac14}}$ pair 930.5 $\alpha = n + \frac12$ pair 930.6 $\alpha = \frac12, 1, \frac32$ pairs 915.4, 872, 915.7	$\dfrac{(c^2-g^2)^{\frac12\alpha-\frac12}J_{\alpha-1}[\lambda(c^2-g^2)^{\frac12}]}{(2\pi)^{\frac12}c^{\alpha-\frac12}\lambda^{\alpha-1}}$, $\lvert g\rvert < c$
931.2	$(p+\rho)^{\frac12}\exp(2\sigma^2 p)K_{\nu+\frac12}[\sigma^2(p+\rho)]$ $\times K_{\nu-\frac12}[\sigma^2(p+\rho)]$ $\nu = \pm\frac14, \pm\frac34$ pairs 912.2, 912.6 $\sigma = \infty$ pair 526 $\sigma = 0$ pair 524.2 $\frac12 \leqq \lvert R(\nu)\rvert$ $R(\rho) = 0$	$\dfrac{2^{\frac12}\pi^{\frac32}\exp[-\rho(g+2\sigma^2)]}{g^{\frac12}(g+2\sigma^2)^{\frac12}(g+4\sigma^2)^{\frac12}}$ $\times\cosh\left[2\nu\cosh^{-1}\left(\dfrac{g+2\sigma^2}{2\sigma^2}\right)\right]$ $=\dfrac{\pi^{\frac32}\exp[-\rho(g+2\sigma^2)]}{2^{\frac12}g^{\frac12}(g+2\sigma^2)^{\frac12}(g+4\sigma^2)^{\frac12}}$ $\times\left\{\left[\dfrac{g^{\frac12}+(g+4\sigma^2)^{\frac12}}{2\sigma}\right]^{4\nu}\right.$ $\left.+\left[\dfrac{g^{\frac12}+(g+4\sigma^2)^{\frac12}}{2\sigma}\right]^{-4\nu}\right\}$, $0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
931.3	$(p+\rho)^{\frac12}K_{\nu+\frac12}[x(p+\rho)]K_{\nu-\frac12}[x(p+\rho)]$ $\boxed{R}$ $x = 0$: pair 524.2 $\boxed{D}$ $\frac34 \leqq \lvert R(\nu)\rvert,\ R(\rho) = 0$	$\dfrac{2^{\frac12}\pi^{\frac12}e^{-\rho g}}{g^{\frac12}(g^2-4x^2)^{\frac12}}\cosh\left[2\nu\cosh^{-1}\left(\dfrac{g}{2x}\right)\right]$ $= \dfrac{\pi^{\frac12}e^{-\rho g}}{2^{\frac12}g^{\frac12}(g^2-4x^2)^{\frac12}}$ $\times\left\{\left[\dfrac{(g-2x)^{\frac12}+(g+2x)^{\frac12}}{2x^{\frac12}}\right]^{4\nu}\right.$ $\left.+\left[\dfrac{(g-2x)^{\frac12}+(g+2x)^{\frac12}}{2x^{\frac12}}\right]^{-4\nu}\right\},$ $2x < g$
933.2	$(p+\lambda)^{\frac12}I_v[a(p+\lambda)]K_{v+\frac12}[a(p+\lambda)]$ $\boxed{S}$ $v = -1, 0$: pairs 909.91, 914.61 $\boxed{B}$ $a = \infty$: pair 526	$\dfrac{(-1)^v e^{-\lambda g}}{(\frac12\pi g)^{\frac12}(4a^2-g^2)^{\frac12}}$ $\times\cos\left[(2v+\tfrac12)\cos^{-1}\left(\dfrac{g}{2a}\right)\right]$ $= \dfrac{(-1)^v e^{-\lambda g}}{(2\pi g)^{\frac12}(4a^2-g^2)^{\frac12}}$ $\times\left\{\left[\dfrac{(2a+g)^{\frac12}+i(2a-g)^{\frac12}}{2a^{\frac12}}\right]^{4v+1}\right.$ $\left.+\left[\dfrac{(2a+g)^{\frac12}-i(2a-g)^{\frac12}}{2a^{\frac12}}\right]^{4v+1}\right\},$ $0 < g < 2a$
937.0	$(p+\lambda)^{\frac12}\{I_{\nu-\frac12}[a(p+\lambda)]I_{-\nu-\frac12}[a(p+\lambda)]$ $-I_{\nu+\frac12}[a(p+\lambda)]I_{-\nu+\frac12}[a(p+\lambda)]\}$ $\boxed{S}$ $\nu = v \pm \frac14$: pair 933.2 $\boxed{S}$ $\nu = \pm\frac14, \pm\frac34$: pairs 914.61, 909.91 $\boxed{B}$ $a = \infty$: pair 526	$\dfrac{2^{\frac32}e^{-\lambda g}}{\pi^{\frac32}g^{\frac12}(4a^2-g^2)^{\frac12}}\cos\left[2\nu\cos^{-1}\left(\dfrac{g}{2a}\right)\right]$ $= \dfrac{2^{\frac12}e^{-\lambda g}}{\pi^{\frac32}g^{\frac12}(4a^2-g^2)^{\frac12}}$ $\times\left\{\left[\dfrac{(g-2a)^{\frac12}+(g+2a)^{\frac12}}{2a^{\frac12}}\right]^{4\nu}\right.$ $\left.+\left[\dfrac{(g-2a)^{\frac12}+(g+2a)^{\frac12}}{2a^{\frac12}}\right]^{-4\nu}\right\},$ $0 < g < 2a$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
938.0 Key	$(p+\rho)^{\frac12}\{J_{\nu+\frac12}[\sigma(p+\rho)]Y_{\nu-\frac12}[\sigma(p+\rho)]$ $- J_{\nu-\frac12}[\sigma(p+\rho)]Y_{\nu+\frac12}[\sigma(p+\rho)]\}$ $\nu = -\frac32, -\frac12, \frac12, \frac32$ pairs 938.5, 938.2, 938.3, 938.6 $\sigma = \infty$ pair 526 $\sigma = 0$ pair 524.2 $R(\nu) \leqq -\frac12$, $R(\rho) = 0$	$\dfrac{(2\sigma)^{\frac12} e^{-\rho g}[g + (g^2 + 4\sigma^2)^{\frac12}]^{-2\nu}}{(\frac12\pi)^{\frac12} g^{\frac12}(g^2 + 4\sigma^2)^{\frac12}}$	$0 < g$
938.2	$\sin[\sigma(p+\rho)]J_0[\sigma(p+\rho)]$ $- \cos[\sigma(p+\rho)]Y_0[\sigma(p+\rho)]$ $\sigma = \infty$ pair 526 $\sigma = 0$	$\dfrac{2^{\frac12} e^{-\rho g}[g + (g^2 + 4\sigma^2)^{\frac12}]^{\frac12}}{\pi g^{\frac12}(g^2 + 4\sigma^2)^{\frac12}}$	$0 < g$
938.3	$\cos[\sigma(p+\rho)]J_0[\sigma(p+\rho)]$ $+ \sin[\sigma(p+\rho)]Y_0[\sigma(p+\rho)]$ $\sigma = \infty$ pair 526 $\sigma = 0$	$\dfrac{2^{\frac12}\sigma e^{-\rho g}[g + (g^2 + 4\sigma^2)^{\frac12}]^{-\frac12}}{\pi g^{\frac12}(g^2 + 4\sigma^2)^{\frac12}}$	$0 < g$
938.5	$\sin[\sigma(p+\beta)]J_1[\sigma(p+\beta)]$ $- \cos[\sigma(p+\beta)]Y_1[\sigma(p+\beta)]$ $\sigma = 0$ pair 438 $\sigma = \infty$ pair 526	$\dfrac{e^{-\beta g}[g + (g^2 + 4\sigma^2)^{\frac12}]^{\frac32}}{2^{\frac32}\pi\sigma g^{\frac12}(g^2 + 4\sigma^2)^{\frac12}}$	$0 < g$
938.6	$\cos[\sigma(p+\rho)]J_1[\sigma(p+\rho)]$ $+ \sin[\sigma(p+\rho)]Y_1[\sigma(p+\rho)]$ $\sigma = \infty$ pair 526 $\sigma = 0$	$-\dfrac{2^{\frac32}\sigma^2 e^{-\rho g}[g + (g^2 + 4\sigma^2)^{\frac12}]^{-\frac32}}{\pi g^{\frac12}(g^2 + 4\sigma^2)^{\frac12}}$	$0 < g$
939.3	$I_{\alpha-1}\{c[(p+\lambda)^{\frac12} + (p+\mu)^{\frac12}]^2\}$ $\times I_{\alpha-1}\{c[(p+\lambda)^{\frac12} - (p+\mu)^{\frac12}]^2\}$ $\alpha = 1$ pair 871.3 $\lambda = \mu$ pair 914.2	$\dfrac{e^{-\frac12(\lambda+\mu)g}J_{2\alpha-2}[\frac12(\lambda-\mu)(16c^2 - g^2)^{\frac12}]}{\pi(16c^2 - g^2)^{\frac12}}$	$\lvert g\rvert < 4c$

No.	Coefficient $F(f)$	Coefficient $G(g)$
940.1 Key	$\exp(4\tau^2 p) I_{\alpha-\frac{1}{2}}\{\tau^2[(p+\rho)^{\frac{1}{2}} - (p+\sigma)^{\frac{1}{2}}]^2\}$ $\times K_{\alpha-\frac{1}{2}}\{\tau^2[(p+\rho)^{\frac{1}{2}} + (p+\sigma)^{\frac{1}{2}}]^2\}$ $\boxed{S}$ $\alpha = 1$: pair 859.1 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 817 $\boxed{B}$ $\tau = \infty$: pair 926.0 $\boxed{R}$ $\rho = \sigma$: pair 911.1 $\boxed{R}$ $\tau = 0$: pair 576.1 $\boxed{C}$ α an integer; τ unrestricted by notation	$\dfrac{\exp[-\frac{1}{2}(\rho+\sigma)(g+4\tau^2)]}{g^{\frac{1}{2}}(g+8\tau^2)^{\frac{1}{2}}}$ $\times I_{2\alpha-1}[\frac{1}{2}(\rho-\sigma)g^{\frac{1}{2}}(g+8\tau^2)^{\frac{1}{2}}]$, $0 < g$
940.3	$I_{\alpha-\frac{1}{2}}\{x[(p+\rho)^{\frac{1}{2}} - (p+\sigma)^{\frac{1}{2}}]^2\}$ $\times K_{\alpha-\frac{1}{2}}\{x[(p+\rho)^{\frac{1}{2}} + (p+\sigma)^{\frac{1}{2}}]^2\}$ $\boxed{S}$ $\alpha = 1$: pair 863.1 $\boxed{B}$ $\rho = \infty$ or $\sigma = \infty$: pair 817 $\boxed{R}$ $x = 0$: pair 576.1 $\boxed{R}$ $\rho = \sigma$: pair 911.6	$\dfrac{e^{-\frac{1}{2}(\rho+\sigma)g} I_{2\alpha-1}[\frac{1}{2}(\rho-\sigma)(g^2-16x^2)^{\frac{1}{2}}]}{(g^2-16x^2)^{\frac{1}{2}}}$, $4x < g$
941.0	$(\rho^2 - p^2)^{\frac{1}{2}\nu} K_\nu[\sigma(\rho^2-p^2)^{\frac{1}{2}}]$ $\boxed{S}$ $\nu = -\frac{3}{2}, -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 1$ pairs 867.5, 917.5, 868, 916.5, 917, 867, 917.8 $\boxed{B}$ $\rho = \infty$ and $\sigma = \infty$: pair 710.0 $\boxed{R}$ $\rho = 0$: pair 918.5 $\boxed{R}$ $\sigma = 0$: pair 569.0 $\boxed{D}$ $R(\nu) \leqq -1$, $R(\rho) = 0$ $\boxed{D}$ $\frac{1}{2} \leqq R(\nu)$, $R(\sigma) = 0$	$\dfrac{\sigma^\nu \rho^{\nu+\frac{1}{2}} K_{\nu+\frac{1}{2}}[\rho(g^2+\sigma^2)^{\frac{1}{2}}]}{(2\pi)^{\frac{1}{2}}(g^2+\sigma^2)^{\frac{1}{2}\nu+\frac{1}{4}}}$
942.0	$\dfrac{J_{\frac{1}{2}-\alpha}[x(\rho^2-p^2)^{\frac{1}{2}}]}{(\rho^2-p^2)^{\frac{1}{2}\alpha-\frac{1}{4}}}$ $\boxed{S}$ $\alpha = \frac{1}{2}, 1, \frac{3}{2}$: pairs 915.4, 871.5, 915.7 $\boxed{B}$ $x = \infty$ and $\rho = \infty$: pair 710.0 $\boxed{R}$ $x = 0$: pair 569.0 $\boxed{R}$ $\alpha = n + \frac{1}{2}$: pair 930.6 $\boxed{R}$ $\rho = 0$: pair 919.5 $\boxed{D}$ $\frac{3}{2} \leqq R(\alpha)$, $R(\rho) = 0$	$\begin{cases} -\dfrac{(x^2-g^2)^{\frac{1}{2}\alpha-\frac{1}{4}} Y_{1-\alpha}[\rho(x^2-g^2)^{\frac{1}{2}}]}{(2\pi)^{\frac{1}{2}} x^{\alpha-\frac{1}{2}} \rho^{\alpha-1}}, & \lvert g\rvert < \lvert x\rvert \\ -\dfrac{2^{\frac{1}{2}} \cos \pi\alpha}{\pi^{\frac{3}{2}} x^{\alpha-\frac{1}{2}} \rho^{\alpha-1}} (g^2-x^2)^{\frac{1}{2}\alpha-\frac{1}{4}} \times K_{1-\alpha}[\rho(g^2-x^2)^{\frac{1}{2}}], & \lvert x\rvert < \lvert g\rvert \end{cases}$

No	Coefficient $F(p)$	Coefficient $G(g)$
945 1 Key	$K_\nu[(\sigma^{\frac12}+\tau^{\frac12})(p+\rho)^{\frac12}]I_\nu[(\sigma^{\frac12}-\tau^{\frac12})(p+\rho)^{\frac12}]$ $\nu = \frac12$ pair 823.3 $\sigma = 0$ or $\tau = 0$ pair 945 5 $\sigma = \infty$ or $\tau = \infty$ pair 823 $\sigma = \tau$ pair 927 0 $R(\nu) \leqq -1$ $R(\rho) = 0$	$\frac{1}{2g}\exp\left(-\rho g - \frac{\sigma+\tau}{2g}\right)I_\nu\left(\frac{\sigma-\tau}{2g}\right)$ $\quad 0 < g$
945 5	$K_\nu[\sigma^{\frac12}(p+\rho)^{\frac12}]I_\nu[\sigma^{\frac12}(p+\rho)^{\frac12}]$ $\nu = \frac12$ pair 823 5 $\sigma = \infty$ pair 526 $\sigma = 0$ pair 524 2 $R(\nu) \leqq -1$ $R(\rho) = 0$	$\frac{1}{2g}\exp\left(-\rho g - \frac{\sigma}{2g}\right)I_\nu\left(\frac{\sigma}{2g}\right)$ $\quad 0 < g$
951 0	$\exp[\frac14\sigma(p+\lambda)^2]D_{-\alpha}[\sigma^{\frac12}(p+\lambda)]$ $\alpha = \frac12$ 1 2 pairs 923 1 903 0 903 3 $\sigma = \infty$ pair 524 2 $\sigma = 0$ $R(\lambda) < 0$ $R(\sigma) = 0$ $2 \leqq R(\alpha)$ $R(\lambda) = 0$ $R(\sigma) = 0$	$\frac{g^{\alpha-1}}{\sigma^{\frac12\alpha}\Gamma(\alpha)}\exp\left(-\lambda g - \frac{g^2}{2\sigma}\right)$ $\quad 0 < g$
951 1	$\frac{1}{p+\beta}\exp[\frac14\sigma(p+\beta)^2]D_{1-\alpha}[\sigma^{\frac12}(p+\beta)]$ $\alpha = \frac32$ 2 3 pairs 923 2 903 1 903 5 $\alpha = 1$ pair 438 $\sigma = 0$ pair 438 $\sigma = \infty$ pair 524 2	$\frac{2^{\frac12\alpha-1}}{\Gamma(\alpha-1)}e^{-\beta g}\gamma\left(\frac12\alpha-\frac12, \frac{g^2}{2\sigma}\right)$ $\quad 0 < g$
952 0	$\frac{1}{(p+\rho)^{\frac12\alpha}}\exp\left[\frac{1}{4\lambda(p+\rho)}\right] \times D_{-\alpha}\left[\frac{1}{\lambda^{\frac12}(p+\rho)^{\frac12}}\right]$ $\alpha = \frac12$ 1 2 3 pairs 924 3 905 0 902 5 902 8 $\rho = 0$ pair 952 1 $\lambda = \infty$ pair 524 2 $\lambda = 0$ $R(\lambda^{\frac12}) < 0$ $R(\rho) = 0$ $\frac12 \leqq R(\alpha)$ [$2 \leqq R(\alpha)$ for transposed pair] $R(\rho) = 0$ $R(\lambda^{\frac12}) = 0$	$\frac{1}{\Gamma(\alpha)}(2g)^{\frac12\alpha-1}\exp\left(-\rho g - \frac{2^{\frac12}g^{\frac12}}{\lambda^{\frac12}}\right)$ $\quad 0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
952.1	$\frac{1}{p^{\frac{1}{2}\alpha}}\exp\left(\frac{1}{4\sigma^2 p}\right)D_{-\alpha}\left(\frac{1}{\sigma p^{\frac{1}{2}}}\right)$ $\boxed{S}$ $\alpha = \frac{1}{2}, 1, 2$: pairs 924.4, 902.1, 902.6 $\boxed{B}$ $\sigma = \infty$: pair 521 $\boxed{D}$ $\sigma = 0$ $\boxed{D}$ $\frac{5}{2} \leqq R(\alpha)$ [$2 \leqq R(\alpha)$ for transposed pair], $R(\sigma) = 0$	$\frac{1}{\Gamma(\alpha)}(2g)^{\frac{1}{2}\alpha-1}\exp\left(-\frac{2^{\frac{1}{2}}g^{\frac{1}{2}}}{\sigma}\right)$,	$0 < g$
952.3	$\frac{1}{(p+\rho)^{\frac{1}{2}\alpha-\frac{1}{2}}}\exp\left[\frac{1}{4\lambda(p+\rho)}\right] \times \left\{D_{1-\alpha}\left[\frac{-1}{\lambda^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}\right] - D_{1-\alpha}\left[\frac{1}{\lambda^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}\right]\right\}$ $\boxed{S}$ $\alpha = 1, \frac{3}{2}, 2, 3$: pairs 904.2, 926.3, 904.1, 653 $\boxed{S}$ $\rho = 0$: pair 952.5 $\boxed{B}$ $\lambda = \infty$: pair 524.2 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$ $\boxed{D}$ $\frac{7}{2} \leqq R(\alpha)$ [$3 \leqq R(\alpha)$ for transposed pair], $R(\rho) = 0$	$\frac{2^{\frac{1}{2}\alpha-1}}{\Gamma(\alpha-1)}e^{-\rho g}g^{\frac{1}{2}\alpha-\frac{3}{2}}\sinh\left(\frac{2^{\frac{1}{2}}g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
952.4	$\frac{1}{(p+\rho)^{\frac{1}{2}\alpha}}\exp\left[\frac{1}{4\lambda(p+\rho)}\right] \times \left\{D_{-\alpha}\left[\frac{-1}{\lambda^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}\right] + D_{-\alpha}\left[\frac{1}{\lambda^{\frac{1}{2}}(p+\rho)^{\frac{1}{2}}}\right]\right\}$ $\boxed{S}$ $\alpha = \frac{1}{2}, 1, 2$: pairs 926.6, 651, 904.5 $\boxed{S}$ $\rho = 0$: pair 952.6 $\boxed{B}$ $\lambda = \infty$: pair 524.2 $\boxed{D}$ $\lambda = 0$ $\boxed{D}$ $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$ $\boxed{D}$ $\frac{5}{2} \leqq R(\alpha)$ [$2 \leqq R(\alpha)$ for transposed pair], $R(\rho) = 0$	$\frac{2^{\frac{1}{2}\alpha}}{\Gamma(\alpha)}e^{-\rho g}g^{\frac{1}{2}\alpha-1}\cosh\left(\frac{2^{\frac{1}{2}}g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$	
952.5	$\frac{1}{p^{\frac12\alpha-\frac12}}\exp\left(-\frac{1}{4cp}\right)\left[D_{\alpha-2}\left(-\frac{1}{c^{\frac12}p^{\frac12}}\right) - D_{\alpha-2}\left(\frac{1}{c^{\frac12}p^{\frac12}}\right)\right]$ $\alpha = 1, \frac32, 3$ pairs 904.4, 926.4, 653.4 $c = \infty$ pair 521 $\frac32 \leqq R(\alpha)$ [$3 \leqq R(\alpha)$ for transposed pair]	$\frac{2^{\frac12\alpha}\sin(\frac12\pi\alpha)}{\pi^{\frac12}}g^{\frac12\alpha-1}\sin\left(\frac{2^{\frac12}g^{\frac12}}{c^{\frac12}}\right),$	$0 < g$
952.6	$\frac{1}{p^{\frac12\alpha}}\exp\left(-\frac{1}{4cp}\right)\left[D_{\alpha-1}\left(\frac{1}{c^{\frac12}p^{\frac12}}\right) + D_{\alpha-1}\left(-\frac{1}{c^{\frac12}p^{\frac12}}\right)\right]$ $\alpha = \frac12, 1$ pairs 926.7, 652.2 $c = \infty$ pair 521 $\frac12 \leqq R(\alpha)$ [$2 \leqq R(\alpha)$ for transposed pair]	$\frac{2^{\frac12\alpha+\frac12}\sin(\frac12\pi\alpha)}{\pi^{\frac12}}g^{\frac12\alpha-1}\cos\left(\frac{2^{\frac12}g^{\frac12}}{c^{\frac12}}\right)$	$0 < g$
953.1	$\exp(\frac14\sigma^2 p)D_{-\alpha}[\sigma(p+\rho)^{\frac12}]$ $\alpha = \frac12, 1$ pairs 911.3, 907.1 $\sigma = \infty$ pair 524.2 $\sigma = 0$ $1 \leqq R(\alpha)$, $R(\sigma) = 0$	$\frac{\sigma g^{\frac12\alpha-1}\exp[-\rho(g+\frac14\sigma^2)]}{2^{\frac12\alpha+\frac12}\Gamma(\frac12\alpha)(g+\frac14\sigma^2)^{\frac12\alpha+\frac12}}$	$0 < g$
953.3	$\frac{1}{(p+\rho)^{\frac12}}\exp(\frac14\sigma^2 p)D_{1-\alpha}[\sigma(p+\rho)^{\frac12}]$ $\alpha = \frac32, 2$ pairs 911.4, 906.1 $\alpha = 1$ pair 526 $\sigma = 0$ pair 526 $\sigma = \infty$ pair 524.2 $3 \leqq R(\alpha)$, $R(\sigma) = 0$	$\frac{g^{\frac12\alpha-1}\exp[-\rho(g+\frac14\sigma^2)]}{2^{\frac12\alpha-\frac12}\Gamma(\frac12\alpha)(g+\frac14\sigma^2)^{\frac12\alpha-\frac12}},$	$0 < g$
953.7	$D_{-\alpha}[x^{\frac12}(p+\rho)^{\frac12}]$ $\alpha = \frac12$ pair 911.7 $x = 0$ $1 \leqq R(\alpha)$, $x < 0$	$\frac{x^{\frac12}e^{-\rho g}(g-\frac14 x)^{\frac12\alpha-1}}{2^{\frac12\alpha+\frac12}\Gamma(\frac12\alpha)(g+\frac14 x)^{\frac12\alpha+\frac12}}$	$\frac14 x < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
953.9	$\frac{1}{(p+\rho)^{\frac12}} D_{1-\alpha}[x^{\frac12}(p+\rho)^{\frac12}]$ $\boxed{S}$ $\alpha = \frac{3}{2}$: pair 911.8 $\boxed{B}$ $x = 0$: pair 526 $\boxed{D}$ $3 \leqq R(\alpha)$, $x < 0$	$\frac{e^{-\rho g}(g-\frac14 x)^{\frac12\alpha-1}}{2^{\frac12\alpha-\frac12}\Gamma(\frac12\alpha)(g+\frac14 x)^{\frac12\alpha-\frac12}}$,	$\frac14 x < g$
954.1	$D_{\alpha-1}[c^{\frac12}(p+\lambda)^{\frac12}] + D_{\alpha-1}[-c^{\frac12}(p+\lambda)^{\frac12}]$ $\boxed{S}$ $\alpha = \frac12$: pair 914.4 $\boxed{D}$ $1 \leqq R(\alpha)$	$\frac{2^{\frac12\alpha}c^{\frac12}\sin(\frac12\pi\alpha)e^{-\lambda g}(\frac14 c+g)^{\frac12\alpha-1}}{\Gamma(\frac12-\frac12\alpha)(\frac14 c-g)^{\frac12\alpha+\frac12}}$,	$\lvert g\rvert < \frac14 c$
954.5	$\frac{1}{(p+\lambda)^{\frac12}}\{D_{\alpha-2}[-c^{\frac12}(p+\lambda)^{\frac12}]$ $- D_{\alpha-2}[c^{\frac12}(p+\lambda)^{\frac12}]\}$ $\boxed{S}$ $\alpha = \frac32$: pair 914.9 $\boxed{D}$ $3 \leqq R(\alpha)$	$\frac{2^{\frac12\alpha}\sin(\frac12\pi\alpha)e^{-\lambda g}(\frac14 c+g)^{\frac12\alpha-1}}{\Gamma(\frac32-\frac12\alpha)(\frac14 c-g)^{\frac12\alpha-\frac12}}$,	$\lvert g\rvert < \frac14 c$
955.1	$\frac{1}{(p+\rho)^{\nu}}\exp(\sigma^2 p)\Gamma[\nu, \sigma^2(p+\rho)]$ $\boxed{S}$ $\nu = 0, \frac12, 1$: pairs 958.1, 906.1, 438 $\boxed{S}$ $\sigma = 0$: pair 524.2 $\boxed{B}$ $\sigma = \infty$: pair 438 $\boxed{C}$ ν a positive integer; σ unrestricted by notation $\boxed{D}$ $1 \leqq R(\nu)$, $R(\rho) = 0$ $\boxed{D}$ $R(\nu) \leqq 0$, $R(\sigma) = 0$	$\exp[-\rho(g+\sigma^2)](g+\sigma^2)^{\nu-1}$,	$0 < g$
955.3	$\exp(\gamma^2 p)\Gamma[1-\alpha, \gamma^2(p+\rho)]$ $\boxed{S}$ $\alpha = \frac12, 1$: pairs 907.1, 958.1 $\boxed{B}$ $\gamma = 0$: pair 524.2 $\boxed{B}$ $\gamma = \infty$: pair 524.2 $\boxed{D}$ $2 \leqq R(\alpha)$, $R(\rho) = 0$	$\frac{g^{\alpha-1}\exp[-\rho(g+\gamma^2)]}{\gamma^{2\alpha-2}\Gamma(\alpha)(g+\gamma^2)}$,	$0 < g$
955.5	$\frac{\gamma[\alpha, c(p+\lambda)]}{(p+\lambda)^{\alpha}}$ $\boxed{S}$ $\alpha = \frac12, 1$: pairs 907.7, 603.1 $\boxed{B}$ $c = \infty$: pair 524.2	$g^{\alpha-1}e^{-\lambda g}$,	$0 < g < c$

No	Coefficient $F(f)$	Coefficient $G(g)$	
955 6	$\dfrac{\gamma[\alpha\ \ ic(p+\lambda)]}{(p+\lambda)^{\alpha}}$ $\alpha = \frac{1}{2}\ 1$ pairs 907.8 603 1 $c = \infty$ pair 525 2	$-g^{\alpha-1}e^{-\lambda g}$	$-c < g < 0$
955 7	$\dfrac{1}{(p+\rho)^{\nu}}\Gamma[\nu\ \ x(p+\rho)]$ $\nu = -\frac{1}{2}\ 0\ \frac{1}{2}$ pairs 908 3 958 3 906 3 $x = 0$ pair 524 2 $R(\nu) \leqq 0\ \ x < 0$ $1 \leqq R(\nu)\ \ R(\rho) = 0$	$g^{\nu-1}e^{-\rho g}$,	$x < g$
955 9	$\Gamma[1-\alpha\ \ r(p+\rho)]$ $\alpha = \frac{1}{2}\ 1\ \frac{3}{2}$ pairs 907 3 958 3 907 5 $r = 0$ pair 524 2 $2 \leqq R(\alpha)\ \ R(\rho) = 0$	$\dfrac{e^{-\rho g}(g-r)^{\alpha-1}}{\Gamma(\alpha)r^{\alpha-1}g}$,	$r < g$
956 1	$\dfrac{1}{(p+\rho)^{\alpha}}\exp\left[\dfrac{1}{\lambda(p+\rho)}\right] \times \Gamma\left[1-\alpha\ \ \dfrac{1}{\lambda(p+\rho)}\right]$ $\alpha = \frac{1}{2}\ 1\ \frac{3}{2}$ pairs 905 0 959 1 902 5 $\rho = 0$ pair 956 5 $\lambda = \infty$ pair 524 2 $\lambda = 0$ $R(\lambda^{\frac{1}{2}}) < 0\ \ R(\rho) = 0\ \ (\alpha - \frac{1}{2})$ an integer $\pi < \lvert\arg\lambda\rvert\ \ R(\rho) = 0\ \ (\alpha - \frac{1}{2})$ not an integer $2 \leqq R(\alpha)$ [$\frac{1}{2} \leqq R(\alpha)$ for transposed pair] $R(\rho) = 0\ \ R(\lambda^{\frac{1}{2}}) = 0$	$\dfrac{2}{\Gamma(\alpha)}(\lambda g)^{\frac{1}{2}\alpha-1}e^{-\rho g}K_{\alpha-1}\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
956.2	$(p+\rho)^{\nu-1}\exp\left[\dfrac{1}{\lambda(p+\rho)}\right] \times \gamma\left[\nu, \dfrac{1}{\lambda(p+\rho)}\right]$ [S] $\nu = -\frac{1}{2}, 0, \frac{1}{2}, 1$: pairs 904.5, 655.1, 904.1, 654.2 [S] $\rho = 0$: pair 956.7 [B] $\lambda = \infty$: pair 438 [D] $\lambda = 0$ [D] $\lambda \neq -\lvert\lambda\rvert$, $R(\rho) = 0$ [D] $R(\nu) \leqq -1$ [$R(\nu) \leqq -\frac{1}{2}$ for transposed pair], $R(\rho) = 0$	$\dfrac{\Gamma(\nu)}{(\lambda g)^{\frac{1}{2}\nu}} e^{-\rho g} I_\nu\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$, $\quad 0 < g$
956.3	$\gamma\left[\alpha, \dfrac{1}{\lambda(p+\rho)}\right]$ [S] $\alpha = \frac{1}{2}, 1$: pairs 904.2, 654.2 [S] $\rho = 0$: pair 956.6 [B] $\lambda = \infty$: pair 524.2 [D] $\lambda = 0$ [D] $\lambda \neq \lvert\lambda\rvert$, $R(\rho) = 0$ [D] $3 \leqq R(\alpha)$ [$\frac{5}{2} \leqq R(\alpha)$ for transposed pair], $R(\rho) = 0$	$\dfrac{1}{\lambda^{\frac{1}{2}\alpha}} e^{-\rho g} g^{\frac{1}{2}\alpha-1} J_\alpha\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$, $\quad 0 < g$
956.5	$\dfrac{1}{p^\alpha}\exp\left(\dfrac{1}{\sigma^2 p}\right)\Gamma\left(1-\alpha, \dfrac{1}{\sigma^2 p}\right)$ [S] $\alpha = \frac{1}{2}, 1, \frac{3}{2}$: pairs 902.1, 959.2, 902.6 [B] $\sigma = \infty$: pair 521 [D] $\sigma = 0$ [D] $2 \leqq R(\alpha)$ [$\frac{3}{2} \leqq R(\alpha)$ for transposed pair], $R(\sigma) = 0$	$\dfrac{2\sigma^{\alpha-1}}{\Gamma(\alpha)} g^{\frac{1}{2}\alpha-\frac{1}{2}} K_{\alpha-1}\left(\dfrac{2g^{\frac{1}{2}}}{\sigma}\right)$, $\quad 0 < g$
956.6	$\gamma\left(\alpha, \dfrac{1}{cp}\right)$ [S] $\alpha = \frac{1}{2}, 1$: pairs 904.4, 654.3 [B] $c = \infty$: pair 521 [D] $3 \leqq R(\alpha)$ [$\frac{5}{2} \leqq R(\alpha)$ for transposed pair]	$\dfrac{1}{c^{\frac{1}{2}\alpha}} g^{\frac{1}{2}\alpha-1} J_\alpha\left(\dfrac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$, $\quad 0 < g$

No	Coefficient $F(f)$	Coefficient $G(g)$
•956 7	$f^{2\alpha-2}p^{\alpha-2}\exp\left(-\frac{1}{cp}\right)\gamma\left(\alpha-1,\frac{1}{f^2cp}\right)$ ▣ $\alpha = 1, 2$ pairs 655 2, 654 3 ▣ $R(\alpha) \leqq \frac{1}{2}$ for transposed pair	$\frac{\Gamma(\alpha-1)}{(cg)^{\frac{1}{2}\alpha-\frac{1}{2}}}J_{\alpha-1}\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$, $0 < g$
956 8	$p^{\alpha-2}\exp\left(\frac{1}{cp}\right)\gamma\left(\alpha-1,\frac{1}{cp}\right)$ ▣ $\alpha = \frac{1}{2}, 1, \frac{3}{2}, 2$ pairs 904 1, 655 3, 904 31, 654 6 ▣ $R(\alpha) \leqq \frac{1}{2}$ for transposed pair	$-\frac{\Gamma(\alpha-1)}{(cg)^{\frac{1}{2}\alpha-\frac{1}{2}}}I_{\alpha-1}\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right)$, $g < 0$
957 1	$\frac{1}{(p+\rho)^{2\nu}}\exp[\sigma(p+\rho)^2]\Gamma[\nu,\sigma(p+\rho)^2]$ ▣ $\nu = -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, 1$ pairs 903 3, 959 5, 957 2, 903 1, 442 ▣ $\sigma = \infty$ pair 442 ▣ $\sigma = 0$ pair 524 2 ▣ ν a positive integer, σ unrestricted by notation ▣ $(\frac{1}{2}-\nu)$ a positive integer, $R(\sigma) \neq 0$, ρ unrestricted by notation ▣ $\frac{1}{2} \leqq R(\nu)$, $R(\rho) = 0$ ▣ $R(\nu) \leqq -\frac{1}{2}$, $R(\rho) = 0$, $R(\sigma) = 0$, ν not a positive integer	$\frac{2^{\frac{1}{2}}\sigma^{\nu-\frac{1}{2}}}{g^{\frac{1}{2}}}\exp\left(-\rho g-\frac{g^2}{8\sigma}\right)M_{\frac{1}{4}-\nu,\frac{1}{4}}\left(\frac{g^2}{4\sigma}\right)$ $=\frac{2^{\nu-1}\sigma^{\nu-\frac{1}{2}}\Gamma(\nu)}{\pi^{\frac{1}{2}}}\exp\left(-\rho g-\frac{g^2}{8\sigma}\right)$ $\times\left[D_{-2\nu}\left(\frac{-g}{2^{\frac{1}{2}}\sigma^{\frac{1}{2}}}\right)-D_{-2\nu}\left(\frac{g}{2^{\frac{1}{2}}\sigma^{\frac{1}{2}}}\right)\right]$ $0 < g$
957 2	$\frac{1}{(p+\rho)^{\frac{1}{2}}}\exp[\sigma(p+\rho)^2]\Gamma[\frac{1}{4},\sigma(p+\rho)^2]$ ▣ $\sigma = \infty$ pair 442 ▣ $\sigma = 0$ pair 526	$\frac{\Gamma(\frac{1}{4})}{2\sigma^{\frac{1}{4}}}g^{\frac{1}{2}}\exp\left(-\rho g-\frac{g^2}{8\sigma}\right)I_{\frac{1}{4}}\left(\frac{g^2}{8\sigma}\right)$, $0 < g$
957 3	$\exp[\sigma(p+\rho)^2]\Gamma[1-\alpha,\sigma(p+\rho)^2]$ ▣ $\alpha = 1, \frac{3}{2}$ pairs 959 5, 903 5 ▣ $\alpha = \frac{1}{2}$ pair 903 0 ▣ $\sigma = \infty$ pair 524 2 ▣ $\sigma = 0$ pair 524 2 ▣ $\frac{1}{2} \leqq R(\alpha)$, $R(\rho) = 0$	$\frac{2^{2\alpha-3}}{g^{2\alpha-1}\sigma^{\frac{1}{2}}\Gamma(2\alpha-1)}\exp\left(-\rho g-\frac{g^2}{4\sigma}\right)$ $\times\gamma\left(\alpha-\frac{1}{2},\frac{g^2}{4\sigma}\right)$, $0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$	
957.5	$\exp(-\sigma^3p^2)\Gamma(\frac{1}{4}, \sigma^3\lvert p\rvert^2)$ $\boxed{S}$ $\sigma = 1/(4\pi)^{\frac{1}{3}}$: pair 957.55 ⊡ $\sigma = 0$	$\dfrac{1}{2\pi^{\frac{1}{2}}\sigma^{\frac{3}{2}}}\exp\left(\dfrac{g^2}{4\sigma^3}\right)\Gamma\left(\frac{1}{4}, \dfrac{\lvert g\rvert^2}{4\sigma^3}\right)$	
957.55	$\exp(\pi f^2)\Gamma(\frac{1}{4}, \pi\lvert f\rvert^2)$	$\exp(\pi g^2)\Gamma(\frac{1}{4}, \pi\lvert g\rvert^2)$	
958.1	$\exp(\gamma^2 p)\mathrm{Ei}[\gamma^2(p + \rho)]$ $\boxed{B}$ $\gamma = \infty$: pair 438	$\dfrac{\exp[-\rho(g + \gamma^2)]}{g + \gamma^2}$,	$0 < g$
958.3	$\mathrm{Ei}[a(p + \rho)]$	$\dfrac{1}{g}e^{-\rho a}$,	$a < g$
959.1	$\dfrac{1}{p + \rho}\exp\left[\dfrac{1}{\lambda(p + \rho)}\right]\mathrm{Ei}\left[\dfrac{1}{\lambda(p + \rho)}\right]$ $\boxed{S}$ $\rho = 0$: pair 959.2 $\boxed{B}$ $\lambda = \infty$: pair 438 ⊡ $\lambda = 0$ ⊡ $\pi < \lvert\arg\lambda\rvert$, $R(\rho) = 0$	$2e^{-\rho g}K_0\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$,	$0 < g$
959.2	$\dfrac{1}{p}\exp\left(\dfrac{1}{\sigma^2 p}\right)\mathrm{Ei}\left(\dfrac{1}{\sigma^2 p}\right)$ ⊡ $\sigma = 0$	$2K_0\left(\dfrac{2g^{\frac{1}{2}}}{\sigma}\right)$,	$0 < g$
959.5	$\exp[\sigma(p + \rho)^2]\mathrm{Ei}[\sigma(p + \rho)^2]$ $\boxed{B}$ $\sigma = \infty$: pair 442 ⊡ $\sigma = 0$	$\dfrac{\pi^{\frac{1}{2}}}{i\sigma^{\frac{1}{2}}}\exp\left(-\rho g - \dfrac{g^2}{4\sigma}\right)\mathrm{erf}\left(\dfrac{ig}{2\sigma^{\frac{1}{2}}}\right)$,	$0 < g$
961.1 Key	$\dfrac{1}{(p + \rho)^\nu}\exp(\frac{1}{2}\sigma^2 p)W_{\nu-\alpha, \nu-\frac{1}{2}}[\sigma^2(p + \rho)]$ $\boxed{S}$ $\alpha = 1$: pair 955.1 $\boxed{S}$ $\nu = \frac{1}{4}, \frac{3}{4}$: pairs 953.1, 953.3 $\boxed{S}$ $\nu = \frac{1}{2}\alpha$: pair 955.3 $\boxed{S}$ $\nu = \frac{1}{2}\alpha + \frac{1}{2}$: pair 524.2 $\boxed{S}$ $\nu = \alpha$: pair 911.1 $\boxed{B}$ $\sigma = 0$: pair 524.2 $\boxed{B}$ $\sigma = \infty$: pair 524.2 $\boxed{C}$ $(2\nu - \alpha)$ a positive integer; σ unrestricted by notation ⊡ $1 \leqq R(\nu)$, $R(\rho) = 0$ ⊡ $R(2\nu) \leqq R(\alpha)$, $R(\sigma) = 0$	$\dfrac{\exp[-\rho(g + \frac{1}{2}\sigma^2)]}{\sigma^{2\nu-2}\Gamma(\alpha)}g^{\alpha-1}(g + \sigma^2)^{2\nu-\alpha-1}$,	$0 < g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
961.3	$\frac{1}{(p+\lambda)^{\alpha+\beta}} M_{\alpha-\beta,\,\alpha+\beta-\frac{1}{2}}[c(p+\lambda)]$ $\boxed{S}$ $\alpha=\beta$ pair 914.2 $\boxed{S}$ $\alpha+\beta=\frac{1}{2}, \frac{3}{2}$ pairs 954.1, 954.5	$\frac{\Gamma(2\alpha+2\beta)e^{-\lambda g}}{\Gamma(2\alpha)\Gamma(2\beta)c^{\alpha+\beta-1}}(\frac{1}{2}c+g)^{2\alpha-1}$ $\times(\frac{1}{2}c-g)^{2\beta-1}, \quad \lvert g\rvert<\frac{1}{2}c$
961.5	$\frac{1}{(p+\rho)^{\nu}} W_{\nu-\alpha,\,\nu-\frac{1}{2}}[x(p+\rho)]$ $\boxed{S}$ $\nu=\frac{1}{2}, \frac{3}{2}$ pairs 953.7, 953.9 $\boxed{S}$ $\nu=\alpha$ pair 911.6 $\boxed{E}$ $x=0$ pair 524.2 $\boxed{D}$ $1\leqq R(\nu)$, $R(\rho)=0$ $\boxed{D}$ $R(2\nu)\leqq R(\alpha)$, $x<0$	$\frac{1}{x^{\nu-1}\Gamma(\alpha)}e^{-\rho g}(g-\frac{1}{2}x)^{\alpha-1}(g+\frac{1}{2}x)^{2\nu-\alpha-1},$ $\frac{1}{2}x<g$
963.1 Key	$\frac{1}{(p+\rho)^{\alpha-\nu}}\exp\left[\frac{1}{2\lambda(p+\rho)}\right]$ $\times M_{\nu-\alpha,\,\nu-\frac{1}{2}}\left[\frac{1}{\lambda(p+\rho)}\right]$ $\boxed{S}$ $\alpha=1$ pair 956.2 $\boxed{S}$ $\nu=\frac{1}{2}, \frac{3}{2}$ pairs 952.4, 952.3 $\boxed{S}$ $\nu=\frac{1}{2}\alpha$ pair 650.0 $\boxed{S}$ $\nu=\frac{1}{2}\alpha+\frac{1}{2}$ pair 956.3 $\boxed{S}$ $\nu=\alpha$ pair 926.0 $\boxed{S}$ $\rho=0$ pair 963.3 $\boxed{E}$ $\lambda=\infty$ pair 524.2 $\boxed{E}$ $\lambda=0$ $\boxed{D}$ $\lambda\neq-\lvert\lambda\rvert$, $R(\rho)=0$ $\boxed{D}$ $R(\nu+1)\leqq R(\alpha)$ [$R(\nu+\frac{1}{2})\leqq R(\alpha)$ for transposed pair], $R(\rho)=0$	$\frac{\Gamma(2\nu)}{\lambda^{\frac{1}{2}}\Gamma(\alpha)}e^{-\rho g}g^{\alpha-\nu-\frac{1}{2}}J_{2\nu-1}\left(\frac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right), \quad 0<g$
963.3	$p^{\beta-1}\exp\left(-\frac{1}{2cp}\right)M_{1-\beta,\,\alpha+\beta-\frac{3}{2}}\left(\frac{1}{cp}\right)$ $\boxed{S}$ $\alpha=1$ pair 956.7 $\boxed{S}$ $\beta=1$ pair 925.1 $\boxed{S}$ $\frac{1}{2}\alpha+\beta=1, \frac{3}{2}$ pairs 650.4, 956.6 $\boxed{S}$ $\alpha+\beta=\frac{5}{4}, \frac{7}{4}$ pairs 952.6, 952.5 $\boxed{E}$ $c=\infty$ pair 521 $\boxed{D}$ $R(\beta)\leqq\frac{1}{4}$ for transposed pair	$\frac{\Gamma(2\alpha+2\beta-2)}{c^{\frac{1}{2}}\Gamma(\alpha)g^{\beta-\frac{1}{2}}}J_{2\alpha+2\beta-3}\left(\frac{2g^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right),$ $0<g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
964.1	$\dfrac{1}{(p+\rho)^{\alpha+\beta-\frac{1}{2}}}\exp\left[\dfrac{1}{2\lambda(p+\rho)}\right] \times W_{\frac{1}{2}-\alpha-\beta,\,\alpha-\beta}\left[\dfrac{1}{\lambda(p+\rho)}\right]$ $\boxed{S}$ $\alpha=\frac{1}{2}$ or $\beta=\frac{1}{2}$: pair 956.1 $\boxed{S}$ $\alpha=\beta$: pair 964.6 $\boxed{S}$ $\alpha+\beta=\frac{1}{2}$: pair 924.1 $\boxed{S}$ $\alpha-\beta=\pm\frac{1}{4}$: pair 952.0 $\boxed{S}$ $\rho=0$: pair 964 3 $\boxed{E}$ $\lambda=\infty$: pair 524.2 $\boxed{D}$ $\lambda=0$ $\boxed{D}$ $R(\lambda^{\frac{1}{2}})<0$, $R(\rho)=0$, $(2\alpha-2\beta+\frac{1}{2})$ an integer $\boxed{D}$ $\pi<\lvert\arg\lambda\rvert$, $R(\rho)=0$, $(2\alpha-2\beta+\frac{1}{2})$ not an integer. $\boxed{D}$ $\frac{3}{2}\leqq R(\alpha+\beta)$ [$\frac{5}{4}\leqq R(\alpha+\beta)$ for transposed pair], $R(\rho)=0$, $R(\lambda^{\frac{1}{2}})=0$	$\dfrac{2}{\lambda^{\frac{1}{2}}\Gamma(2\alpha)\Gamma(2\beta)}e^{-\rho g}g^{\alpha+\beta-1}K_{2\alpha-2\beta}\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$, $0<g$
964.3	$\dfrac{1}{p^{\alpha+\beta-\frac{1}{2}}}\exp\left(\dfrac{1}{2\sigma^2 p}\right)W_{\frac{1}{2}-\alpha-\beta,\,\alpha-\beta}\left(\dfrac{1}{\sigma^2 p}\right)$ $\boxed{S}$ $\alpha=\frac{1}{2}$ or $\beta=\frac{1}{2}$: pair 956.5 $\boxed{S}$ $\alpha=\beta$: pair 964.7 $\boxed{S}$ $\alpha+\beta=\frac{1}{2}$: pair 924.2 $\boxed{S}$ $\alpha-\beta=\pm\frac{1}{4}$: pair 952.1 $\boxed{E}$ $\sigma=\infty$: pair 521 $\boxed{D}$ $\sigma=0$ $\boxed{D}$ $\frac{1}{2}\leqq R(\alpha+\beta)$ [$\frac{5}{4}\leqq R(\alpha+\beta)$ for transposed pair], $R(\sigma)=0$	$\dfrac{2}{\sigma\Gamma(2\alpha)\Gamma(2\beta)}g^{\alpha+\beta-1}K_{2\alpha-2\beta}\left(\dfrac{2g^{\frac{1}{2}}}{\sigma}\right)$, $0<g$
964.6	$\dfrac{1}{(p+\rho)^{\alpha-\frac{1}{2}}}\exp\left[\dfrac{1}{2\lambda(p+\rho)}\right] \times W_{\frac{1}{2}-\alpha,\,0}\left[\dfrac{1}{\lambda(p+\rho)}\right]$ $\boxed{S}$ $\alpha=\frac{1}{2}, 1$: pairs 924.5, 959.1 $\boxed{S}$ $\rho=0$: pair 964.7 $\boxed{E}$ $\lambda=\infty$: pair 524 2 $\boxed{D}$ $\lambda=0$ $\boxed{D}$ $\pi<\lvert\arg\lambda\rvert$, $R(\rho)=0$ $\boxed{D}$ $\frac{3}{4}\leqq R(\alpha)$ [$\frac{5}{4}\leqq R(\alpha)$ for transposed pair], $R(\rho)=0$, $R(\lambda^{\frac{1}{2}})=0$	$\dfrac{2}{\lambda^{\frac{1}{2}}[\Gamma(\alpha)]^2}e^{-\rho g}g^{\alpha-1}K_0\left(\dfrac{2g^{\frac{1}{2}}}{\lambda^{\frac{1}{2}}}\right)$, $0<g$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
964 7	$\frac{1}{p^{\alpha-\frac{1}{2}}}\exp\left(\frac{1}{2\sigma^2 p}\right) W_{\frac{1}{2}-\alpha,0}\left(\frac{1}{\sigma^2 p}\right)$ $\boxed{S}$ $\alpha = \frac{1}{2}, 1$ pairs 924 6, 959 2 $\boxed{L}$ $\sigma = \infty$ pair 521 $\boxed{L}$ $\sigma = 0$ $\boxed{C}$ $\frac{1}{2} \leqq R(\alpha)$ [$\frac{1}{2} \leqq R(\alpha)$ for transposed pair] $R(\sigma) = 0$	$\frac{2}{\sigma[\Gamma(\alpha)]^2} g^{\alpha-1} K_0\left(\frac{2g^{\frac{1}{2}}}{\sigma}\right)$, $0 < g$
965 1 Key	$\frac{1}{(p+\rho)^{2\nu}}\exp[\frac{1}{2}\sigma(p+\rho)^2]$ $\times W_{\nu-\alpha,\nu-\frac{1}{2}}[\sigma(p+\rho)^2]$ $\boxed{S}$ $\alpha = 1$ pair 957 1 $\boxed{S}$ $\nu = \frac{1}{4}\ \frac{3}{4}$ pairs 951 0, 951 1 $\boxed{S}$ $\nu = \frac{3}{4}\alpha + \frac{1}{4}$ pair 965 6 $\boxed{S}$ $\nu = \frac{1}{2}\alpha$ pair 957 3 $\boxed{S}$ $\nu = \frac{1}{2}\alpha + \frac{1}{2}$ pair 524 2 $\boxed{S}$ $\nu = \alpha$ pair 923 6 $\boxed{L}$ $\sigma = 0$ pair 524 2 $\boxed{L}$ $\sigma = \infty$ pair 524 2 $\boxed{C}$ $(2\nu - \alpha)$ a positive integer, σ unrestricted by notation $\boxed{C}$ $(\frac{1}{2} - 2\nu)$ a positive integer, $R(\sigma) \neq 0$, ρ unrestricted by notation $\boxed{C}$ $\frac{3}{4} \leqq R(\nu)$, $R(\rho) = 0$ $\boxed{C}$ $R(2\nu + \frac{1}{2}) \leqq R(\alpha)$, $R(\rho) = 0$ $R(\sigma) = 0$, $(2\nu - \alpha)$ not a positive integer	$\frac{2^{\alpha+\frac{1}{2}}\sigma^{\nu-\frac{1}{2}\alpha+\frac{1}{2}}}{\Gamma(2\alpha)} g^{\alpha-1}\exp\left(-\rho g - \frac{g^2}{8\sigma}\right)$ $\times M_{\frac{1}{2}-2\nu+\frac{1}{2}\alpha,\frac{1}{2}\alpha-\frac{1}{2}}\left(\frac{g^2}{4\sigma}\right)$, $0 < g$
965 6	$\frac{1}{(p+\rho)^{2\alpha+\frac{1}{2}}}\exp[\frac{1}{2}\sigma(p+\rho)^2]$ $\times W_{\frac{1}{4}-\frac{3}{2}\alpha,\alpha-\frac{1}{4}}[\sigma(p+\rho)^2]$ $\boxed{S}$ $\alpha = \frac{1}{2}\ \frac{1}{4}\ \frac{3}{4}$ pairs 923 4, 957 2, 903 5 $\boxed{L}$ $\sigma = \infty$ pair 524 2 $\boxed{L}$ $\sigma = 0$ pair 524 2 $\boxed{C}$ $\frac{3}{8} \leqq R(\alpha)$, $R(\rho) = 0$	$\frac{2^{3\alpha-2}\Gamma(2\alpha+\frac{3}{4})}{\sigma^{\alpha-\frac{1}{4}}\Gamma(8\alpha)} g^{2\alpha-\frac{1}{2}}\exp\left(-\rho g - \frac{g^2}{8\sigma}\right)$ $\times I_{2\alpha-\frac{1}{2}}\left(\frac{g^2}{8\sigma}\right)$, $0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
966.2	$\lvert p\rvert^{2\alpha-\frac{1}{2}}\exp(-\frac{1}{2}\sigma^3p^2)$ $\times W_{\frac{1}{4}-3\alpha,\frac{1}{4}-\alpha}(\sigma^3\lvert p\rvert^2)$ $\boxed{S}$ $\sigma = 1/(4\pi)^{\frac{1}{2}}$: pair 966.25 $\boxed{S}$ $\alpha = \frac{1}{4}, \frac{3}{8}$: pairs 923.5, 957.5 $\boxed{B}$ $\alpha = \frac{1}{8}$: pair 523.1 $\boxed{B}$ $\sigma = \infty$: pair 522.5 $\boxed{R}$ $\alpha = \frac{1}{2}$: pair 903.4 $\boxed{R}$ $\sigma = 0$: pair 522.5 $\boxed{D}$ $\frac{1}{8} \leqq R(\alpha)$, $R(\sigma) = 0$	$\dfrac{\lvert g\rvert^{2\alpha-\frac{1}{2}}}{2^{2\alpha-\frac{1}{2}}\pi^{\frac{1}{2}}\sigma^{6\alpha-3}}\exp\left(\dfrac{g^2}{8\sigma^3}\right)$ $\times W_{\frac{1}{4}-3\alpha,\frac{1}{4}-\alpha}\left(\dfrac{\lvert g\rvert^2}{4\sigma^3}\right)$
966.25	$\lvert f\rvert^{2\alpha-\frac{1}{2}}\exp(\frac{1}{2}\pi f^2)W_{\frac{1}{4}-3\alpha,\frac{1}{4}-\alpha}(\pi\lvert f\rvert^2)$ $\boxed{S}$ $\alpha = \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}$: pairs 523, 923.55, 957.55, 903.45	$\lvert g\rvert^{2\alpha-\frac{1}{2}}\exp(\frac{1}{2}\pi g^2)W_{\frac{1}{4}-3\alpha,\frac{1}{4}-\alpha}(\pi\lvert g\rvert^2)$
971.1 Key	$\zeta[\alpha+1, \gamma(p+\beta)]$ $\boxed{S}$ $\alpha = k$: pair 975.1 $\boxed{S}$ $\alpha = 1$: pair 976.1 $\boxed{B}$ $\gamma = 0$: pair 524.2 $\boxed{B}$ $\gamma = \infty$: pair 524.2	$\dfrac{g^\alpha e^{-\beta g}}{\gamma^{\alpha+1}\Gamma(\alpha+1)\left[1-\exp\left(-\dfrac{g}{\gamma}\right)\right]}$, $0 < g$
973.1 Key	$\Gamma(p+\beta+1)\zeta(p+\beta+1)$	$\dfrac{e^{-(\beta+1)g}}{\exp(e^{-g})-1}$
975.1	$\psi^{(k)}[\gamma(p+\beta)]$ $\boxed{S}$ $k = 1$: pair 976.1 $\boxed{B}$ $\gamma = 0$: pair 431 $\boxed{B}$ $\gamma = \infty$: pair 431	$\dfrac{(-1)^{k-1}g^k e^{-\beta g}}{\gamma^{k+1}\left[1-\exp\left(-\dfrac{g}{\gamma}\right)\right]}$, $0 < g$
976.1	$\psi'[\gamma(p+\beta)]$ $\boxed{B}$ $\gamma = 0$: pair 442 $\boxed{B}$ $\gamma = \infty$: pair 438	$\dfrac{g e^{-\beta g}}{\gamma^2\left[1-\exp\left(-\dfrac{g}{\gamma}\right)\right]}$, $0 < g$
981	$\mathfrak{S}_0(f) = \dfrac{1}{\pi}\lim_{\beta\to 0}\left(\dfrac{\beta}{\beta^2+f^2}\right)$	1
982	$\mathfrak{S}_0(f-f_0) = \dfrac{1}{\pi}\lim_{\beta\to 0}\left[\dfrac{\beta}{\beta^2+(f-f_0)^2}\right]$ $\boxed{S}$ $f_0 = 0$: pair 981	$\mathrm{cis}(2\pi f_0 g)$

No	Coefficient $F(f)$	Coefficient $G(g)$
983	$\mathfrak{S}_0(f-f_0)+\mathfrak{S}_0(f+f_0)$ $=\frac{1}{\pi}\lim_{\beta\to 0}\left[\frac{\beta}{\beta^2+(f-f_0)^2}+\frac{\beta}{\beta^2+(f+f_0)^2}\right]$ ■ $f_0=0$ pair 981	$2\cos(2\pi f_0 g)$
984	$\mathfrak{S}_0(f-f_0)-\mathfrak{S}_0(f+f_0)$ $=\frac{1}{\pi}\lim_{\beta\to 0}\left[\frac{\beta}{\beta^2+(f-f_0)^2}-\frac{\beta}{\beta^2+(f+f_0)^2}\right]$	$i2\sin(2\pi f_0 g)$
985	$\mathfrak{S}_1(f)=\lim_{\beta\to 0}\left[-\frac{2\pi f}{\beta^{\frac{3}{2}}}\exp\left(-\frac{\pi f^2}{\beta}\right)\right]$	$-i2\pi g$
986	$\mathfrak{S}_{-1}(f)=\lim_{\beta\to 0}\left[\left(\lambda+\frac{\lvert f\rvert}{2f}\right)e^{-\beta\lvert f\rvert}\right]$	$-\frac{1}{i2\pi g}+\lambda\mathfrak{S}_0(g)$
987	$\mathfrak{S}_{-2}(f)=\lim_{\beta\to 0}\left[(\tfrac{1}{2}\lvert f\rvert+\lambda f+\mu)e^{-\beta\lvert f\rvert}\right]$	$-\frac{1}{4\pi^2 g^2}+\frac{\lambda}{i2\pi}\mathfrak{S}_1(g)+\mu\mathfrak{S}_0(g)$

Part 10 Fourier Series for Coefficients $G(g)$ with Period 2π

$F(f)$ is a limit which vanishes for all values of f excepting only certain of the points $f=\nu/2\pi$, $\nu=0$, ± 1, ± 2, ..., for which $F(f)$ becomes infinite and covers the finite area $A_\nu\lambda^\nu=\frac{1}{2\pi}\int_{-\pi}^{\pi}G(g)e^{-i\nu g}dg$, as stipulated below. Both A_ν and λ may be complex.

1001	$1,\ \frac{\lambda}{1!},\ \frac{\lambda^2}{2!},\ \frac{\lambda^3}{3!},\ \cdots,\ \frac{\lambda^n}{n!},\ \cdots$ ■ $z=\lambda e^{ig}$	e^z $=e^{\lambda\cos g}\cos(\lambda\sin g)+ie^{\lambda\cos g}\sin(\lambda\sin g)$ $=\cosh z+\sinh z$ $=\cosh(\lambda\cos g)\cos(\lambda\sin g)$ $+i\sinh(\lambda\cos g)\sin(\lambda\sin g)$ $+\sinh(\lambda\cos g)\cos(\lambda\sin g)$ $+i\cosh(\lambda\cos g)\sin(\lambda\sin g)$ $=\frac{1}{2}(\cosh z+\cos z)$ $+\frac{1}{2}(\sinh z+\sin z)$ $+\frac{1}{2}(\cosh z-\cos z)$ $+\frac{1}{2}(\sinh z-\sin z)$

No.	Coefficient $F(f)$	Coefficient $G(g)$
1002	$0, \dfrac{\lambda}{1}, \dfrac{\lambda^2}{2}, \dfrac{\lambda^3}{3}, \cdots, \dfrac{\lambda^n}{n}, \cdots$ $\blacksquare$ $z = \lambda e^{ig} = re^{ig+j\theta}, j = (-1)^{\frac{1}{2}}$ $\boxtimes$ $1 < \lvert\lambda\rvert$	$-\log(1-z)$ $= -\frac{1}{2}\log(1-2\lambda\cos g+\lambda^2) + i\tan^{-1}\dfrac{\lambda\sin g}{1-\lambda\cos g}$ $= -\frac{1}{2}\log(1-z^2) + \tanh^{-1}z$ $= -\frac{1}{4}\log(1-2\lambda^2\cos 2g+\lambda^4) + i\frac{1}{2}\tan^{-1}\dfrac{\lambda^2\sin 2g}{1-\lambda^2\cos 2g} + \frac{1}{2}\tanh^{-1}\dfrac{2\lambda\cos g}{1+\lambda^2} + i\frac{1}{2}\tan^{-1}\dfrac{2\lambda\sin g}{1-\lambda^2}$ $= -\frac{1}{4}\log[(1-r^2)^2+4r(r\cos g - \cos\theta)(\cos g - r\cos\theta)] + i\frac{1}{2}\tan^{-1}\dfrac{2r\sin g(\cos\theta - r\cos g)}{1-2r\cos g\cos\theta+r^2\cos 2g} + j\frac{1}{2}\tan^{-1}\dfrac{2r\sin\theta(\cos g - r\cos\theta)}{1-2r\cos g\cos\theta+r^2\cos 2\theta} + ij\frac{1}{2}\tanh^{-1}\dfrac{2r\sin g\sin\theta}{1-2r\cos g\cos\theta+r^2}$
1003	$1, \lambda, \lambda^2, \cdots, \lambda^n, \cdots$ $\blacksquare$ $\Delta = (1-r^2)^2 + 4r(\cos g - r\cos\theta)(r\cos g - \cos\theta)$ $\blacksquare$ $z = \lambda e^{ig} = re^{j\theta+ig}, j = (-1)^{\frac{1}{2}}$ $\boxdot$ $1 \leqq \lvert\lambda\rvert$	$\dfrac{1}{1-z}$ $= \dfrac{1-\lambda\cos g}{1-2\lambda\cos g+\lambda^2} + i\dfrac{\lambda\sin g}{1-2\lambda\cos g+\lambda^2}$ $= \dfrac{1}{1-z^2} + \dfrac{z}{1-z^2}$ $= \dfrac{1}{1-z^k} + \dfrac{z}{1-z^k} + \dfrac{z^2}{1-z^k} + \cdots + \dfrac{z^{k-1}}{1-z^k}$ $= \dfrac{\Delta + (1-r^2)(1-2r\cos g\cos\theta+r^2)}{2\Delta} + i\dfrac{r\sin g[(1+r^2)\cos\theta - 2r\cos g]}{\Delta} + j\dfrac{r\sin\theta[(1+r^2)\cos g - 2r\cos\theta]}{\Delta} + ij\dfrac{r(1-r^2)\sin g\sin\theta}{\Delta}$

No	Coefficient $F(f)$	Coefficient $G(g)$	
1004	$1,\ \frac{1}{2}\lambda,\ \frac{1\cdot 3}{2\cdot 4}\lambda^2,\ \cdots,\ \frac{(2n)!}{4^n(n!)^2}\lambda^n,\ \cdots$ $\boxed{R}$ $z = \lambda e^{ig}$ $\boxed{C}$ $1 < \lvert\lambda\rvert$	$\frac{1}{(1-z)^{1/2}}$ $= \left[\frac{1-\lambda\cos g+(1-2\lambda\cos g+\lambda^2)^{1/2}}{2(1-2\lambda\cos g+\lambda^2)}\right]^{1/2}$ $+ i\left[\frac{\lambda\cos g-1+(1-2\lambda\cos g+\lambda^2)^{1/2}}{2(1-2\lambda\cos g+\lambda^2)}\right]^{1/2}$ $= \frac{\cos\frac{1}{4}(\pi-g)}{[2\sin\frac{1}{2}g]^{1/2}} + i\frac{\sin\frac{1}{4}(\pi-g)}{[2\sin\frac{1}{2}g]^{1/2}}$ if $\lambda = 1$	

Part 11 Contour Integrals Paths Parallel Real Axis

A pair $F(f)$ $G(g)$ with integration limits $f = \pm\infty - ix/(2\pi)$ in Part 11 is the same as a pair $F\left(f-\frac{ix}{2\pi}\right)$ $e^{-xg}G(g)$ in Parts 4–10. The transposed pair is $e^{-xf}G(f)$ $F\left(-g+\frac{ix}{2\pi}\right)$ with the path of integration along the real axis of f.

1101	$\frac{1}{p^k}$ $\boxed{P}$ $c - i\infty$ to $c + i\infty$ $\boxed{A}$ pair 431	$\frac{g^{k-1}}{(k-1)!}$,	$0 < g$
1102	$\frac{1}{(p-\lambda)^k}$ $\boxed{P}$ $x - i\infty$ to $x + i\infty$, $R(\lambda) < x$ $\boxed{A}$ pair 431 $\boxed{S}$ $\lambda = 0$, pair 1101 $\boxed{S}$ $x = 0$, pair 431	$\frac{e^{\lambda g}g^{k-1}}{(k-1)!}$,	$0 < g$
1104	$\frac{1}{p^2-\lambda^2}$ $\boxed{P}$ $c - i\infty$ to $c + i\infty$, $\lvert R(\lambda)\rvert < c$ $\boxed{A}$ pair 448.1 $\boxed{S}$ $\lambda = 0$, pair 1101 with $k = 2$	$\frac{1}{\lambda}\sinh\lambda g$,	$0 < g$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$	
1105	$\dfrac{p}{p^2 - \lambda^2}$ P $c - i\|\infty\|$ to $c + i\|\infty\|$; $\|R(\lambda)\| < c$ A pair 449.1 with pair 448.1 S $\lambda = 0$: pair 1101 with $k = 1$	$\cosh \lambda g,$	$0 < g$
1106	$\dfrac{1}{(p - \lambda)(p - \mu)}$ P $x - i\|\infty\|$ to $x + i\|\infty\|$; $R(\lambda) < x$, $R(\mu) < x$ A pair 448 S $\mu = \lambda$: pair 1102 with $k = 2$ S $\mu = -\lambda$: pair 1104 S $x = 0$: pair 448 B $\lambda = \infty$ or $\mu = \infty$: pair 1102 with $k = 1$	$\dfrac{e^{\lambda g} - e^{\mu g}}{\lambda - \mu},$	$0 < g$
1107	$\dfrac{p}{(p - \lambda)(p - \mu)}$ P $x - i\|\infty\|$ to $x + i\|\infty\|$; $R(\lambda) < x$, $R(\mu) < x$ A pair 449 with pair 448 S $\lambda = 0$ or $\mu = 0$: pair 1102 with $k = 1$ S $\mu = -\lambda$: pair 1105 S $x = 0$: pair 449	$\dfrac{\lambda e^{\lambda g} - \mu e^{\mu g}}{\lambda - \mu},$	$0 < g$
1108	$\dfrac{1}{p^{\alpha}}$ P $c - i\|\infty\|$ to $c + i\|\infty\|$ A pair 524.2 S $\alpha = k$: pair 1101 B $c = 0$: pair 521	$\dfrac{g^{\alpha-1}}{\Gamma(\alpha)},$	$0 < g$

No	Coefficient $F(f)$	Coefficient $G(g)$	
1109	$\dfrac{1}{(p-\lambda)^{\alpha}}$ $x - i\|\infty\|$ to $x + i\|\infty\|$ $R(\lambda) < x$ pair 524.2 $\lambda = 0$ pair 1108 $\alpha = k$ pair 1102 $x = 0$ pair 524.2 $R(\alpha) < 1$, $R(\lambda) = x$	$\dfrac{g^{\alpha-1}e^{\lambda g}}{\Gamma(\alpha)}$,	$0 < g$
1110	$\dfrac{1}{(p^2-\lambda^2)^{\frac{1}{2}}}$ $r - i\|\infty\|$ to $r + i\|\infty\|$ $\|R(\lambda)\| \leqq r$ pair 555 $\lambda = 0$ pair 1101 with $k = 1$ $r = 0$ pair 557	$I_0(\lambda g)$,	$0 < g$
1111	$\dfrac{1}{(p^2-\lambda^2)^{\frac{3}{2}}}$ $c - i\|\infty\|$ to $c + i\|\infty\|$ $\|R(\lambda)\| < c$ pair 570.7 $\lambda = 0$ pair 1101 with $k = 3$	$\dfrac{g}{\lambda} I_1(\lambda g)$,	$0 < g$
1116	$\log\left(\dfrac{p-\mu}{p-\lambda}\right)$ $x - i\|\infty\|$ to $x + i\|\infty\|$ $R(\lambda) \leqq x$ $R(\mu) \leqq x$ pair 894 $\mu = \lambda$ pair 1102 with $k = 1$ $x = 0$ pair 894	$\dfrac{e^{\lambda g} - e^{\mu g}}{g}$,	$0 < g$

No.	Coefficient $F(f)$	Coefficient $G(g)$
	Part 12. Contour Integrals. Closed Paths	
	The letters f, g and p are complex quantities not restricted as in the table of notation. These pairs cannot be transposed by pair (217).	
1251	$\dfrac{1}{p^k}$ $\boxed{\text{P}}$ $(0+)$ $\boxed{\text{S}}$ $k = 1$: pair 1252	$\dfrac{g^{k-1}}{(k-1)!}$
1252	$\dfrac{1}{p}$ $\boxed{\text{P}}$ $(0+)$	1
1253	$\dfrac{1}{(p-\lambda)^k}$ $\boxed{\text{P}}$ $(\lambda+)$ $\boxed{\text{S}}$ $\lambda = 0$: pair 1251 $\boxed{\text{S}}$ $k = 1$: pair 1254	$\dfrac{g^{k-1}e^{\lambda g}}{(k-1)!}$
1254	$\dfrac{1}{p-\lambda}$ $\boxed{\text{P}}$ $(\lambda+)$ $\boxed{\text{S}}$ $\lambda = 0$: pair 1252	$e^{\lambda g}$
1255	$\dfrac{1}{(p-\lambda)(p-\mu)}$ $\boxed{\text{P}}$ $(\lambda+);\ \mu$ $\boxed{\text{B}}$ $\mu = \infty$: pair 1254 $\boxed{\text{D}}$ $\mu = \lambda$	$\dfrac{e^{\lambda g}}{\lambda-\mu}$
1256	$\dfrac{1}{(p-\lambda)(p-\mu)}$ $\boxed{\text{P}}$ $(\lambda+, \mu+)$ $\boxed{\text{S}}$ $\lambda = \mu$: pair 1253 with $k = 2$ $\boxed{\text{B}}$ $\lambda = \infty$ or $\mu = \infty$: pair 1254	$\dfrac{e^{\lambda g} - e^{\mu g}}{\lambda-\mu}$ $= \dfrac{2}{\lambda-\mu} e^{\frac{1}{2}(\lambda+\mu)g} \sinh[\tfrac{1}{2}(\lambda-\mu)g]$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
1257	$\dfrac{1}{(p-\lambda)(p-\mu)}$ $(\lambda+\ \mu-)$ $\lambda=\infty$ or $\mu=\infty$ pair 1254 $\mu=\lambda$	$\dfrac{e^{\lambda g}+e^{\mu g}}{\lambda-\mu}$ $=\dfrac{2}{\lambda-\mu}\,e^{\frac{1}{2}(\lambda+\mu)g}\cosh[\tfrac{1}{2}(\lambda-\mu)g]$
1258	p^{ν} $(-\ \|\infty\|\cos\theta\ \ 0+)$ $\nu=-k$ pair 1251 ν a positive integer or zero	$\dfrac{1}{\Gamma(-\nu)g^{\nu+1}}$ $G(g)$ is defined only for the domain $\|\arg g+\theta\|<\tfrac{1}{2}\pi$
1259	$(p-\lambda)^{\nu}$ $(-\ \|\infty\|\cos\theta\ \ \lambda+)$ $\lambda=0$ pair 1258 $\nu=-k$ pair 1253 ν a positive integer or zero	$\dfrac{e^{\lambda g}}{\Gamma(-\nu)g^{\nu+1}}$ $G(g)$ is defined only for the domain $\|\arg g+\theta\|<\tfrac{1}{2}\pi$
1260	$(p-\lambda)^{\nu}(p-\mu)^{\nu}$ $(\lambda+\ \mu-)$ $\nu=-1$ pair 1257 $\lambda=\infty$ or $\mu=\infty$ pair 1259 $\mu=\lambda$ ν a positive integer or zero	$\dfrac{\pi^{\frac{1}{2}}}{\Gamma(-\nu)}\left(\dfrac{\lambda-\mu}{g}\right)^{\nu+\frac{1}{2}}e^{\frac{1}{2}(\lambda+\mu)g}I_{\nu+\frac{1}{2}}[\tfrac{1}{2}(\lambda-\mu)g]$
1261	$(p-\lambda)\ (p-\mu)^{\nu}$ $(-\ \|\infty\|\cos\theta\ \ \lambda+)\ \ \mu$ $\nu=-1$ pair 1255 $\mu=\infty$ pair 1259 $\mu=\lambda$ ν a positive integer or zero	$\dfrac{(\mu-\lambda)^{\nu+\frac{1}{2}}e^{\frac{1}{2}(\lambda+\mu)g+i\nu\pi}}{\pi^{\frac{1}{2}}\Gamma(-\nu)g^{\nu+\frac{1}{2}}}K_{\nu+\frac{1}{2}}[\tfrac{1}{2}(\mu-\lambda)g]$ If ν is not an integer $G(g)$ is defined only for the domain $\|\arg g+\theta\|<\tfrac{1}{2}\pi$
1262	$(p-\lambda)^{\nu}(p-\mu)^{\nu}$ $(-\ \|\infty\|\cos\theta\ \ \lambda+\ \mu+)$ $\mu=\lambda$ pair 1259 $\nu=-1$ pair 1256 $\lambda=\infty$ or $\mu=\infty$ pair 1259 ν a positive integer or zero	$\dfrac{\pi^{\frac{1}{2}}}{\Gamma(-\nu)}\left(\dfrac{\lambda-\mu}{g}\right)^{\nu+\frac{1}{2}}e^{\frac{1}{2}(\lambda+\mu)g}I_{-\nu-\frac{1}{2}}[\tfrac{1}{2}(\lambda-\mu)g]$ If ν is not an integer $G(g)$ is defined only for the domain $\|\arg g+\theta\|<\tfrac{1}{2}\pi$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
1263	$(p-\lambda)^{\mu}(p-z)^{\nu}$ $\boxed{P}$ $(-\,\vert\infty\vert\,\mathrm{cis}\,\theta;\lambda+,z+)$ $\boxed{S}$ $\mu=\nu$: pair 1262 $\boxed{S}$ $z=\lambda$: pair 1259 $\boxed{S}$ $\mu=0$ or $\nu=0$: pair 1259 $\boxed{B}$ $z=\infty$ or $\lambda=\infty$: pair 1259 $\boxed{D}$ $\mu+\nu$ a positive integer or zero	$\dfrac{(z-\lambda)^{\frac{1}{2}\mu+\frac{1}{2}\nu}e^{\frac{1}{2}(z+\lambda)g}}{\Gamma(-\mu-\nu)g^{\frac{1}{2}\mu+\frac{1}{2}\nu+1}}$ $\times M_{\frac{1}{2}\nu-\frac{1}{2}\mu,\,-\frac{1}{2}\mu-\frac{1}{2}\nu-\frac{1}{2}}[(z-\lambda)g]$ $\boxed{F}$ If $\mu+\nu$ is not an integer $G(g)$ is defined only for the domain $\vert\arg g+\theta\vert<\frac{1}{2}\pi$
1264	$(p-\lambda)^{\mu}(p-z)^{\nu}$ $\boxed{P}$ $(-\,\vert\infty\vert\,\mathrm{cis}\,\theta;\lambda+);z$ $\boxed{S}$ $\mu=\nu$: pair 1261 $\boxed{B}$ $\nu=0$: pair 1259 $\boxed{B}$ $z=\infty$: pair 1259 $\boxed{D}$ $z=\lambda$ $\boxed{D}$ μ a positive integer or zero	$\dfrac{(z-\lambda)^{\frac{1}{2}\mu+\frac{1}{2}\nu}e^{\frac{1}{2}(z+\lambda)g+i\nu\pi}}{\Gamma(-\mu)g^{\frac{1}{2}\mu+\frac{1}{2}\nu+1}}$ $\times W_{\frac{1}{2}\nu-\frac{1}{2}\mu,\,\frac{1}{2}\mu+\frac{1}{2}\nu+\frac{1}{2}}[(z-\lambda)g]$ $\boxed{F}$ If μ is not an integer $G(g)$ is defined only for the domain $\vert\arg g+\theta\vert<\frac{1}{2}\pi$
1265	$(p-\lambda)^{\nu}\exp\left[\dfrac{-1}{\alpha(p-\lambda)}\right]$ $\boxed{P}$ $(-\,\vert\infty\vert\,\mathrm{cis}\,\theta;\lambda+)$ $\boxed{B}$ $\alpha=\infty$: pair 1259	$\dfrac{e^{\lambda g}}{(\alpha g)^{\frac{1}{2}\nu+\frac{1}{2}}}J_{-\nu-1}\left(\dfrac{2g^{\frac{1}{2}}}{\alpha^{\frac{1}{2}}}\right)$ $\boxed{F}$ $G(g)$ is defined only for the domain $\vert\arg g+\theta\vert<\frac{1}{2}\pi$
1266	$(p-\lambda)^{\nu}\exp[-\mu(p-\lambda)^{2}]$ $\boxed{P}$ $(-\,\vert\infty\vert\,\mathrm{cis}\,\theta;\lambda+)$ $\boxed{S}$ $\mu=0$: pair 1259 $\boxed{D}$ ν a positive integer or zero $\boxed{D}$ $\frac{1}{2}\pi<\vert\arg\mu+2\theta\vert$	$\dfrac{1}{\Gamma(-\nu)(2\mu)^{\frac{1}{2}\nu+\frac{1}{2}}}\exp\left(\lambda g+\dfrac{g^{2}}{8\mu}\right)$ $\times D_{-\nu-1}\left(\dfrac{g}{2^{\frac{1}{2}}\mu^{\frac{1}{2}}}\right)$ $\boxed{F}$ If $\vert\arg\mu+2\theta\vert=\frac{1}{2}\pi$ and ν is not an integer $G(g)$ is defined only for the domain $\vert\arg g+\theta\vert<\frac{1}{2}\pi$
1267	$\dfrac{(p-\lambda)^{\nu}}{1-e^{p-\lambda}}$ $\boxed{P}$ $(-\,\vert\infty\vert\,\mathrm{cis}\,\theta;\lambda+);\lambda\pm 2k\pi i$ $\boxed{D}$ ν a positive integer	$\dfrac{e^{\lambda g}}{\Gamma(-\nu)}\zeta(\nu+1,g)$ $\boxed{F}$ If ν is not an integer or zero $G(g)$ is defined only for the domain $\vert\arg g+\theta\vert<\frac{1}{2}\pi$

TABLE 1 (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
1268	$\log\left(\frac{p-\mu}{p-\lambda}\right)$ $\boxed{P}$ $(\mu +, \lambda +)$ $\boxed{S}$ $\mu = \lambda$ pair 1254	$\frac{1}{g}(e^{\lambda g} - e^{\mu g})$
1269	$\log\left(\frac{p-\mu}{p-\lambda}\right)$ $\boxed{P}$ $(-i\infty\,\mathrm{cis}\,\theta, \lambda +), \mu$ $\boxed{S}$ $\mu = \lambda$	$\frac{e^{\lambda g}}{g}$ $\boxed{E}$ $G(g)$ is defined only for the domain $\lvert\arg g + \theta\rvert < \frac{1}{2}\pi$

Part 13 Contour Integrals Paths with Arbitrary End Points

The evaluated indefinite integral $\int F(f)e^{i2\pi fg}df$ is tabulated in the $G(g)$ column. Arbitrary upper and lower limits of integration f_0 and f are understood. The letters f, g and p are complex quantities not restricted as in the table of notation. These pairs cannot be transposed by pair (217).

No	Coefficient $F(f)$	Coefficient $G(g)$
1300	$F(f)$	$G(f, g) = \int^{f} F(f)\mathrm{cis}(2\pi fg)df$
1301	$F(af)$	$\frac{1}{a}G\left(af, \frac{g}{a}\right)$
1302	$F(f - f_0)$	$\mathrm{cis}(2\pi f_0 g)G(f - f_0, g)$
1303	$\mathrm{cis}(-2\pi fg_0)F(f)$	$G(f, g - g_0)$
1304	$G(f, \lambda)$ $\boxed{E}$ The coefficient $G(f, g)$ must be the indefinite integral of some particular coefficient $F(f)$	$\frac{1}{i2\pi g}[\mathrm{cis}(2\pi fg)G(f, \lambda) - G(f, g + \lambda)]$
1305	$D_p F = \frac{1}{i2\pi} D_f F$	$\frac{\mathrm{cis}(2\pi fg)}{i2\pi} F(f) - gG(f, g)$
1306	$D_p^{-1}F = i2\pi \int_{-\infty}^{f} F df$	$\frac{\mathrm{cis}(2\pi fg)}{g} \int_{-\infty}^{f} F df - \frac{1}{g}G(f, g)$
1307	$F(-f)$	$-G(-f, -g)$

TABLE I (Continued)

No.	Coefficient $F(f)$	Coefficient $G(g)$
1308	p^n $\boxed{\text{S}}$ $n = 0$: pair 1309	$\dfrac{(-1)^n n! e^{pg}}{i2\pi g^{n+1}}\left[1 - pg + \cdots + \dfrac{(-pg)^n}{n!}\right]$
1309	1	$\dfrac{e^{pg}}{i2\pi g}$
1310	$\dfrac{1}{p^k}$ $\boxed{\text{S}}$ $k = 1$: pair 1311	$\dfrac{-g^{k-1}}{i2\pi(k-1)!}\left\{\mathrm{Ei}(-pg) + e^{pg}\left[\dfrac{1}{pg} + \dfrac{1!}{(pg)^2} + \cdots + \dfrac{(k-2)!}{(pg)^{k-1}}\right]\right\}$
1311	$\dfrac{1}{p}$	$-\dfrac{Ei(-pg)}{i2\pi}$
1312	p^ν $\boxed{\text{S}}$ $\nu = n$: pair 1308 $\boxed{\text{S}}$ $\nu = -k$: pair 1310 $\boxed{\text{S}}$ $\nu = -\frac{1}{2}, \frac{1}{2}$: pairs 1313, 1314	$-\dfrac{\Gamma(\nu + 1, -pg)}{i2\pi(-g)^{\nu+1}}$
1313	$\dfrac{1}{p^{\frac{1}{2}}}$	$\dfrac{\mathrm{erfc}(ip^{\frac{1}{2}}g^{\frac{1}{2}})}{2\pi^{\frac{1}{2}}g^{\frac{1}{2}}}$
1314	$p^{\frac{1}{2}}$	$\dfrac{p^{\frac{1}{2}}e^{pg}}{i2\pi g} - \dfrac{1}{4\pi^{\frac{1}{2}}g^{\frac{3}{2}}}\mathrm{erfc}(ip^{\frac{1}{2}}g^{\frac{1}{2}})$
1315	$\cos \nu p$ $\boxed{\text{S}}$ $\nu = 0$: pair 1309	$\dfrac{e^{pg}(g \cos \nu p + \nu \sin \nu p)}{i2\pi(g^2 + \nu^2)}$
1316	$\sin \nu p$ $\boxed{\text{S}}$ $\nu = 0$: pair 1308 with $n = 1$	$\dfrac{e^{pg}(g \sin \nu p - \nu \cos \nu p)}{i2\pi(g^2 + \nu^2)}$
1317	$\exp(\lambda p^2)$ $\boxed{\text{S}}$ $\lambda = 0$: pair 1309	$\dfrac{1}{4\pi^{\frac{1}{2}}\lambda^{\frac{1}{2}}}\exp\left(-\dfrac{g^2}{4\lambda}\right)\mathrm{erfc}\left(ip\lambda^{\frac{1}{2}} + \dfrac{ig}{2\lambda^{\frac{1}{2}}}\right)$

TABLE I (Continued)

No	Coefficient $F(f)$	Coefficient $G(g)$
1318	$p\exp(\lambda p^2)$ $\boxed{S}$ $\lambda = 0$ pair 1308 with $n = 1$	$-\frac{g}{8\pi^{\frac{1}{2}}\lambda^{\frac{3}{2}}}\exp\left(-\frac{g^2}{4\lambda}\right)\operatorname{erfc}\left(ip\lambda^{\frac{1}{2}} + \frac{ig}{2\lambda^{\frac{1}{2}}}\right) + \frac{\exp(pg + \lambda p^2)}{i4\pi\lambda}$
1319	$\exp(\lambda p^{\frac{1}{2}})$ $\boxed{S}$ $\lambda = 0$ pair 1309	$\frac{-\lambda}{4\pi^{\frac{1}{2}}g^{\frac{3}{2}}}\exp\left(-\frac{\lambda^2}{4g}\right)\operatorname{erfc}\left(ig^{\frac{1}{2}}p^{\frac{1}{2}} + \frac{i\lambda}{2g^{\frac{1}{2}}}\right) + \frac{\exp(\lambda p^{\frac{1}{2}} + pg)}{i2\pi g}$
1320	$\frac{1}{p^{\frac{1}{2}}}\exp(\lambda p^{\frac{1}{2}})$ $\boxed{S}$ $\lambda = 0$ pair 1313	$\frac{1}{2\pi^{\frac{1}{2}}g^{\frac{1}{2}}}\exp\left(-\frac{\lambda^2}{4g}\right)\operatorname{erfc}\left(ip^{\frac{1}{2}}g^{\frac{1}{2}} + \frac{i\lambda}{2g^{\frac{1}{2}}}\right)$
1321	$\log p$	$\frac{1}{i2\pi g}[\operatorname{Ei}(-pg) + e^{pg}\log p]$
1322	$\operatorname{erfc}(\lambda^{\frac{1}{2}}p^{\frac{1}{2}})$ $\boxed{S}$ $\lambda = 0$ pair 1309	$\frac{1}{i2\pi g}\left\{e^{pg}\operatorname{erfc}(\lambda^{\frac{1}{2}}p^{\frac{1}{2}}) - \left(\frac{\lambda}{\lambda - g}\right)^{\frac{1}{2}}\operatorname{erfc}[(\lambda - g)^{\frac{1}{2}}p^{\frac{1}{2}}]\right\}$
1323	$\operatorname{E}_1(\lambda p)$ $\boxed{S}$ $\lambda = 0$ pair 1309	$\frac{1}{i2\pi g}\{e^{pg}\operatorname{E}_1(\lambda p) - \operatorname{E}_1[(\lambda - g)p]\}$
1324	$\Gamma(\nu, \lambda p)$ $\boxed{S}$ $\nu = 0, \frac{1}{2}$ pairs 1323, 1322 $\boxed{S}$ $\lambda = 0$ pair 1309	$\frac{1}{i2\pi g}\left\{e^{pg}\Gamma(\nu, \lambda p) - \left(\frac{\lambda}{\lambda - g}\right)^{\nu}\Gamma[\nu, (\lambda - g)p]\right\}$

Explanation of Table II

(Condensed from the text)

This table illustrates the application of the Table of Fourier Integrals to practical problems calling for the determination of transients in time or space. Table II lists 39 physical systems in the second column and three causes in the first row, namely, a unit impulse, a unit step, and a unit cisoid multiplied by a unit step. The mates for these three causes are given by pairs (403.1), (415), and (440). Multiplying these mates by the admittances, given in the second column of Table II, gives coefficients for which the mates are found by the use of Table I in 85 cases out of the total of 117 cases. The number of the pair used for each transient is given in the lower left-hand corner of the square containing the transient. Given the admittances, each of these transients is thus obtained essentially without further analytical work by looking up the proper pair in Table I and substituting the physical notation. Further details will be found in the text.

Section 1

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse $= \mathfrak{S}_0(t)$ Effect $\mathfrak{A}Y(p)$ 403 1
1	$Y(p) = \dfrac{Cp + G}{LC(p - p_1)(p - p_2)}$ Inductance L and resistance R in series with parallel combination of capacity C and conductance G Cause Voltage across terminals Effect Current through network $\left.\begin{matrix} p_1 \\ p_2 \end{matrix}\right\} = \dfrac{-(RC + LG) \pm \Delta}{2LC}$, $\Delta = \sqrt{(RC - LG)^2 - 4LC}$	$\dfrac{1}{\Delta}[(Cp_1 + G)e^{p_1 t} - (Cp_2 + G)e^{p_2 t}]$, $0 < t$ 448 449
2	$Y(p) = Cp + G$ Same as 1 except R = 0, L = 0	$C\mathfrak{S}_1(t) + G\mathfrak{S}_0(t)$ 403 1 404 1
3	$Y(p) = \exp\left[-\dfrac{x}{v}\sqrt{(p + \rho)^2 - \sigma^2}\right]$ Semi infinite smooth line (resistance R inductance L conductance G, and capacity C per unit length) Cause Initial voltage Effect Voltage at distance x from end $v = (LC)^{-\frac{1}{2}}$, $k = (L/C)^{\frac{1}{2}}$, $\alpha = R/(2L)$ $\beta = G/(2C)$, $\rho = \alpha + \beta$, $\sigma = \alpha - \beta$ $z = \sqrt{t^2 - (x/v)^2}$	$\exp\left(-\dfrac{\rho x}{v}\right)\mathfrak{S}_0\left(t - \dfrac{x}{v}\right)$ $+ \dfrac{\sigma x}{vz}e^{-\rho t} I_1(\sigma z)$, $\dfrac{x}{v} < t$ 601 863 1
4	$Y(p) = \dfrac{1}{k}\sqrt{\dfrac{p + \rho - \sigma}{p + \rho + \sigma}}$ $\times \exp\left[-\dfrac{x}{v}\sqrt{(p + \rho)^2 - \sigma^2}\right]$ Same as 3 except Effect Current at distance x from end	$\dfrac{1}{k}\exp\left(-\dfrac{\rho x}{v}\right)\mathfrak{S}_0\left(t - \dfrac{x}{v}\right)$ $+ \dfrac{1}{k}e^{-\rho t}\left[\dfrac{\sigma t}{z} I_1(\sigma z) - \sigma I_0(\sigma z)\right]$, $\dfrac{x}{v} < t$ 601 864 1

Time Variable.

Cause: Unit Step (0, 1) $= \mathfrak{S}_{-1}(t),\ \lambda = \frac{1}{2}$ Effect: $\mathfrak{M}[Y(p)/p]$ 415	Cause: Unit Cisoid × Unit Step (0, 1) $= e^{p_0 t}\mathfrak{S}_{-1}(t),\ \lambda = \frac{1}{2}$ Effect: $\mathfrak{M}[Y(p)/(p - p_0)]$ 440
$\frac{1}{R + G^{-1}} + \frac{Cp_1 + G}{\Delta p_1} e^{p_1 t} - \frac{Cp_2 + G}{\Delta p_2} e^{p_2 t}, \quad 0 < t$ 448, 454, 415	$\frac{(Cp_0 + G)e^{p_0 t}}{LC(p_0 - p_1)(p_0 - p_2)} + \frac{(Cp_1 + G)e^{p_1 t}}{\Delta(p_1 - p_0)} - \frac{(Cp_2 + G)e^{p_2 t}}{\Delta(p_2 - p_0)}, \quad 0 < t$ 452,* 453*
$C\mathfrak{S}_0(t) + G, \quad 0 < t$ 403.1, 415	$C\mathfrak{S}_0(t) + (Cp_0 + G)e^{p_0 t}, \quad 0 < t$ 403.1, 438*

* By substituting $(p_0 - a)$ for p_0, and taking the limit as $a \to 0$.

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\mathfrak{A}Y(p)$
5	$Y(p) = \exp(-y\sqrt{p+2\beta})$ Same as 3, except $L = 0$ $y = x\sqrt{RC}$	$\frac{y}{2t\sqrt{\pi t}}\exp\left(-\frac{y^2}{4t} - 2\beta t\right), \quad 0 < t$ 817
6	$Y(p) = u\sqrt{p+2\beta}\exp(-y\sqrt{p+2\beta})$ Same as 4, except $L = 0$ $y = x\sqrt{RC}, \quad u = \sqrt{C/R}$	$\frac{u(y^2-2t)}{4t^2\sqrt{\pi t}}\exp\left(-\frac{y^2}{4t} - 2\beta t\right) \quad 0 < t$ 820
7	$Y(p) = \frac{\exp(-y\sqrt{p+2\beta})}{u\sqrt{p+2\beta}}$ Same as 3, except $L = 0$ and Cause Initial current $y = x\sqrt{RC}, \quad u = \sqrt{C/R}$.	$\frac{1}{u\sqrt{\pi t}}\exp\left(-\frac{y^2}{4t} - 2\beta t\right), \quad 0 < t$ 823
8	$Y(p) = \frac{1}{k}\sqrt{\frac{p}{p+2\alpha}}$ $\times\exp\left[-\frac{x}{v}\sqrt{p(p+2\alpha)}\right]$ Same as 4 except $G = 0$	$\frac{1}{k}\exp\left(-\frac{\alpha x}{v}\right)\mathfrak{S}_0\left(t - \frac{x}{v}\right)$ $+\frac{1}{k}e^{-\alpha t}\left[\frac{\alpha t}{s}I_1(\alpha s) - \alpha I_0(\alpha s)\right], \frac{x}{v} < t$ 601 862 1
9	$Y(p) = k\sqrt{\frac{p+2\alpha}{p}}$ Same as 3, except $G = 0$, $x = 0$, and Cause Initial current	$k\mathfrak{S}_0(t) + k\alpha e^{-\alpha t}[I_1(\alpha t) + I_0(\alpha t)]$ $0 < t$ 403 1, 553 1

(*Continued*)

Cause: Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid $\times$ Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$\frac{1}{2}\left[\exp(-y\sqrt{2\beta})\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{2\beta t}\right) + \exp(y\sqrt{2\beta})\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{2\beta t}\right)\right], \quad 0 < t$	$\frac{1}{2}e^{p_0 t}\left[\exp(-y\sqrt{2\beta+p_0}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(2\beta+p_0)t}\right) + \exp(y\sqrt{2\beta+p_0}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(2\beta+p_0)t}\right)\right], \quad 0 < t$
818.1, 415	819 *
$\frac{u}{\sqrt{\pi t}}\exp\left(-\frac{y^2}{4t}-2\beta t\right) + \frac{u\sqrt{2\beta}}{2} \times \left[\exp(-y\sqrt{2\beta})\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{2\beta t}\right) - \exp(y\sqrt{2\beta})\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{2\beta t}\right)\right], \quad 0 < t$	$\frac{u}{\sqrt{\pi t}}\exp\left(-\frac{y^2}{4t}-2\beta t\right) + \frac{u\sqrt{2\beta+p_0}}{2}e^{p_0 t}\left[\exp(-y\sqrt{2\beta+p_0}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(2\beta+p_0)t}\right) - \exp(y\sqrt{2\beta+p_0}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(2\beta+p_0)t}\right)\right], \quad 0 < t$
821.1, 415	822 *
$\frac{1}{2u\sqrt{2\beta}}\left[\exp(-y\sqrt{2\beta}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{2\beta t}\right) - \exp(y\sqrt{2\beta}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{2\beta t}\right)\right], \quad 0 < t$	$\frac{e^{p_0 t}}{2u\sqrt{2\beta+p_0}}\left[\exp(-y\sqrt{2\beta+p_0}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{(2\beta+p_0)t}\right) - \exp(y\sqrt{2\beta+p_0}) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{(2\beta+p_0)t}\right)\right], \quad 0 < t$
824.1, 415	825 *
$\frac{1}{k}e^{-\alpha t}I_0(\alpha z), \quad \frac{x}{v} < t$	
861	
$ke^{-\alpha t}[2\alpha t I_1(\alpha t) + (1+2\alpha t)I_0(\alpha t)], \quad 0 < t$	
554	

* By substituting $(p_0 - a)$ for p_0, and taking the limit as $a \to 0$.

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\partial/\partial t\, Y(p)$
10	$Y(p) = \exp\left(-\frac{\rho x}{v} - \frac{x}{v}p\right)$ Same as 3 except $R/L = G/C$	$\exp\left(-\frac{\rho x}{v}\right)\delta_1\left(t - \frac{x}{v}\right)$ 601
11	$Y(p) = \frac{\sqrt{p + 2\alpha}}{\sqrt{p} + \sqrt{p + 2\alpha}}$ Semi infinite smooth line (resistance R inductance L and capacity C per unit length) Cause Voltage applied through resistance $R_0 = \sqrt{L/C}$ Effect Voltage at end of line $\alpha = R/(2L)$	$\frac{1}{2}\delta_1(t) + \frac{1}{2t}e^{-\alpha t}I_1(\alpha t), \qquad 0 < t$ 403 1 559 1
12	$Y(p) = \frac{\exp(-y\sqrt{p})}{1 + \sqrt{p/\lambda}}$ Semi infinite smooth line (resistance R and capacity C per unit length) Cause Voltage applied through resistance R_0 Effect Voltage at distance x from end $y = x\sqrt{RC}, \quad \lambda = R/(CR_0^2)$	$\sqrt{\frac{\lambda}{\pi t}}\exp\left(-\frac{y^2}{4t}\right) - \lambda\exp(y\sqrt{\lambda} + \lambda t)$ $\times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right), \qquad 0 < t$ 809
13	$Y(p) = \frac{u\sqrt{p}\exp(-y\sqrt{p})}{1 + \sqrt{p/\lambda}}$ Same as 12, except Effect Current at distance x from end $u = \sqrt{C/R}$.	$\frac{u(y - 2t\sqrt{\lambda})}{2t}\sqrt{\frac{\lambda}{\pi t}}\exp\left(-\frac{y^2}{4t}\right)$ $+ u\lambda\sqrt{\lambda}\exp(y\sqrt{\lambda} + \lambda t)$ $\times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right), \qquad 0 < t$ 814
14	$Y(p) = \frac{u\sqrt{p}}{1 + \sqrt{p/\lambda}}$ Same as 13, except $x = 0$ $u = \sqrt{C/R}$	$u\sqrt{\lambda}\left[\delta_1(t) - \sqrt{\frac{\lambda}{\pi t}} + \lambda e^{\lambda t}\operatorname{erfc}\sqrt{\lambda t}\right],$ $0 < t$ 403 1 543

(Continued)

Cause: Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid $\times$ Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$\exp\left(-\frac{\rho x}{v}\right), \quad \frac{x}{v} < t$ 602	$\exp\left[-\frac{x}{v}(\rho + p_0) + p_0 t\right], \quad \frac{x}{v} < t$ 604 *
$1 - \frac{1}{2}e^{-\alpha t}[I_0(\alpha t) + I_1(\alpha t)], \quad 0 < t$ 553.1, 415	
$\operatorname{erfc}\frac{y}{2\sqrt{t}} - \exp(y\sqrt{\lambda} + \lambda t) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right), \quad 0 < t$ 811, 415	$\frac{1}{2}e^{p_0 t}\left[\frac{\exp(-y\sqrt{p_0})}{1+\sqrt{p_0/\lambda}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{p_0 t}\right) + \frac{\exp(y\sqrt{p_0})}{1-\sqrt{p_0/\lambda}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{p_0 t}\right)\right] - \frac{1}{1-p_0/\lambda}\exp(y\sqrt{\lambda} + \lambda t) \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right), \quad 0 < t$ 812 *
$u\sqrt{\lambda}\exp(y\sqrt{\lambda} + \lambda t)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right), \quad 0 < t$ 815	$\frac{u\sqrt{p_0}}{2}e^{p_0 t}\left[\frac{\exp(-y\sqrt{p_0})}{1+\sqrt{p_0/\lambda}} \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{p_0 t}\right) - \frac{\exp(y\sqrt{p_0})}{1-\sqrt{p_0/\lambda}} \times \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{p_0 t}\right)\right] + \frac{u\sqrt{\lambda}}{1-p_0/\lambda} \times \exp(y\sqrt{\lambda} + \lambda t)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right), \quad 0 < t$ 816 *
$u\sqrt{\lambda}e^{\lambda t}\operatorname{erfc}\sqrt{\lambda t}, \quad 0 < t$ 551	$\frac{u\sqrt{\lambda}}{1-p_0/\lambda}\left[\sqrt{\frac{p_0}{\lambda}}e^{p_0 t}\operatorname{erf}\sqrt{p_0 t} - \frac{p_0}{\lambda}e^{p_0 t} + e^{\lambda t}\operatorname{erfc}\sqrt{\lambda t}\right], \quad 0 < t$ 552 *

* By substituting $(p_0 - \alpha)$ for p_0, and taking the limit as $\alpha \to 0$.

TABLE II

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\mathcal{M}Y(p)$
15	$Y(p) = \dfrac{C_0 p \exp(-y\sqrt{p})}{1 + \sqrt{p/\mu}}$ Semi infinite smooth line (resistance R and capacity C per unit length) Cause: Voltage applied through capacity C_0 Effect Current at distance x from end $y = x\sqrt{RC}$ $\mu = C/(RC_0^2)$	$\dfrac{C_0(y^2 - 2yt\sqrt{\mu} - 2t + 4\mu t^2)}{4t^2}\sqrt{\dfrac{\mu}{\pi t}}$ $\times \exp\left(-\dfrac{y^2}{4t}\right)$ $- C_0\mu^2 \exp(y\sqrt{\mu} + \mu t)$ $\times \operatorname{erfc}\left(\dfrac{y}{2\sqrt{t}} + \sqrt{\mu t}\right)$ $0 < t$ 810
16	$Y(p) = \dfrac{C_0 p}{1 + \sqrt{p/\mu}}$ Same as 15 except $x = 0$	$- C_0\mu\mathfrak{S}_1(t) + \dfrac{C_0(2\mu t - 1)}{2t}\sqrt{\dfrac{\mu}{\pi t}}$ $- C_0\mu^2 e^{\mu t} \operatorname{erfc}\sqrt{\mu t}$ $0 < t$ 403 1 503 543
17	$Y(p) = \dfrac{w^{2n+1}[\sqrt{(p+\lambda)^2 + w^2} + (p+\lambda)]^{-2n}}{k\sqrt{(p+\lambda)^2 + w^2}}$ Semi infinite artificial line (series element resistance R and inductance L shunt element conductance G and capacity C $R/L = G/C$ mid series termination) Cause Applied voltage Effect Current in nth section $k = (L/C)^{\frac{1}{2}}$ $\lambda = R/L = G/C$ $w = 2(LC)^{-\frac{1}{2}}$	$\dfrac{2}{L}e^{-\lambda t}J_{2n}(wt),$ $0 < t$ 575 1
18	$Y(p) = \dfrac{2(2\alpha)^n}{R}\sqrt{\dfrac{p}{p + 2\alpha}}$ $\times(\sqrt{p + 2\alpha} + \sqrt{p})^{-2n}$ Semi infinite artificial line (series element resistance R shunt element capacity C mid-series termination) Cause Applied voltage Effect Current in nth section $\alpha = 2/(RC)$	$\dfrac{\alpha}{R}e^{-\alpha t}[I_{n-1}(\alpha t) - 2I_n(\alpha t) + I_{n+1}(\alpha t)]$ $0 < t$ 573

(Continued)

Cause: Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid $\times$ Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$C_0\sqrt{\frac{\mu}{\pi t}}\exp\left(-\frac{y^2}{4t}\right) - C_0\mu\exp(y\sqrt{\mu}+\mu t)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\mu t}\right),\quad 0<t$ 809	$\frac{1}{2}C_0p_0\,e^{p_0t}\left[\frac{\exp(-y\sqrt{p_0})}{1+\sqrt{p_0/\mu}}\times\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}-\sqrt{p_0t}\right)+\frac{\exp(y\sqrt{p_0})}{1-\sqrt{p_0/\mu}}\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{p_0t}\right)\right] + C_0\sqrt{\frac{\mu}{\pi t}}\exp\left(-\frac{y^2}{4t}\right)-\frac{C_0\mu}{1-p_0/\mu}\times\exp(y\sqrt{\mu}+\mu t)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}+\sqrt{\mu t}\right),\quad 0<t$ 813 *
$C_0\sqrt{\frac{\mu}{\pi t}}-C_0\mu e^{\mu t}\operatorname{erfc}\sqrt{\mu t},\quad 0<t$ 543	$C_0\sqrt{\frac{\mu}{\pi t}}+\frac{C_0\mu}{1-p_0/\mu}\left[\frac{p_0}{\mu}e^{p_0t}-\frac{p_0}{\mu}\sqrt{\frac{p_0}{\mu}}e^{p_0t}\times\operatorname{erf}\sqrt{p_0t}-e^{\mu t}\operatorname{erfc}\sqrt{\mu t}\right],\quad 0<t$ 545 *
$\frac{2}{R}e^{-\alpha t}I_n(\alpha t),\quad 0<t$ 574	

* By substituting (p_0-a) for p_0, and taking the limit as $a\to 0$.

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\partial\Pi Y(p)$
19	$Y(p) = \exp\left(x - \lvert x\rvert\sqrt{p^2+1}\right)$ Vertical atmospheric waves axis of x vertically upwards velocity $= 1$ height of homogeneous atmosphere $= \frac{1}{2}$ Cause Vertical displacement at $x = 0$ Effect. Vertical displacement at time t of particle whose undisturbed position is x	$e^x \mathbf{1}(t - \lvert x\rvert) - \frac{\lvert x\rvert e^x}{\sqrt{t^2 - x^2}}$ $\times J_1(\sqrt{t^2 - x^2}), \quad \lvert x\rvert < t$ 601 865 1
20	$Y(p) = \frac{\exp\left(x - \lvert x\rvert\sqrt{p^2+1}\right)}{2\sqrt{p^2+1}}$ Same as 19, except Cause Vertical force at $x = 0$	$\frac{e^x}{2} J_0(\sqrt{t^2 - x^2}), \quad \lvert x\rvert < t$ 866
21	$Y(p) = \frac{\lvert y\rvert}{\pi r}\sqrt{p}\,K_1(r\sqrt{p})$ Flow of heat in infinite plane Cause Temperature impulse at origin, temperature maintained zero along x-axis, except at origin Effect Temperature of point with coördinates $(x\ y)$ at time t $r = \sqrt{x^2 + y^2}$	$\frac{\lvert y\rvert}{4\pi t^2}\exp\left(-\frac{r^2}{4t}\right), \quad 0 < t$ 921 1
22	$Y(p) = \frac{1}{p}\left[1 - \exp\left(-y\sqrt{\frac{p}{\nu}}\right)\right]$ Horizontal oscillations of deep viscous fluid, axis of y vertical bottom plane $y = 0$, kinematic coefficient of viscosity $= \nu$ Cause Applied horizontal force Effect Displacement of particle at y at time t, y assumed small	$\operatorname{erf}\frac{y}{2\sqrt{\nu t}}, \quad 0 < t$ 803
23	$Y(p) = \frac{1}{2\pi}K_0\left(\frac{rp}{c}\right)$ Water waves radiating from center in an unlimited sheet of uniform depth h gravity constant $= g$ Cause Pressure at the origin Effect Velocity potential at distance r time t $c^2 = gh$	$\frac{1}{2\pi}\frac{1}{\sqrt{t^2 - \frac{r^2}{c^2}}}, \quad \frac{r}{c} < t$ 912 3

(Continued)

Cause: Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid $\times$ Unit Step (0, 1) Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$\dfrac{\lvert y\rvert}{\pi r^2}\exp\left(-\dfrac{r^2}{4t}\right),\qquad 0<t$ 922.1, 415	
$y\sqrt{\dfrac{t}{\nu\pi}}\exp\left(-\dfrac{y^2}{4\nu t}\right)-\dfrac{y^2}{2\nu}+\left(\dfrac{y^2}{2\nu}+t\right)\operatorname{erf}\dfrac{y}{2\sqrt{\nu t}}.\qquad 0<t$ 804.1, 415	$\dfrac{1}{p_0}e^{p_0t}-\dfrac{1}{p_0}\operatorname{erf}\dfrac{y}{2\sqrt{\nu t}}-\dfrac{e^{p_0t}}{2p_0}\left[\exp\left(-y\sqrt{\dfrac{p_0}{\nu}}\right)\times\operatorname{erfc}\left(\dfrac{y}{2\sqrt{\nu t}}-\sqrt{p_0t}\right)+\exp\left(y\sqrt{\dfrac{p_0}{\nu}}\right)\operatorname{erfc}\left(\dfrac{y}{2\sqrt{\nu t}}+\sqrt{p_0t}\right)\right],$ 805 * $\qquad 0<t$
$\dfrac{1}{2\pi}\cosh^{-1}\dfrac{ct}{r},\qquad \dfrac{r}{c}<t$ 913	

* By substituting (p_0-a) for p_0, and taking the limit as $a\to 0$.

No	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\mathfrak{M}Y(p)$
24	$Y(p) = \frac{\sin[(\pi - y)p]}{\sin \pi p}$ Flow of electricity in thin plane infinite strip axis of x along lower edge of strip axis of y across width of strip $= \pi$ upper edge ($y = \pi$) maintained at zero potential Cause Potential along x axis Effect Potential at point (x y)	$\frac{1}{2\pi} \frac{\sin y}{\cosh x - \cos y}$ 615
25	$Y(p) = \frac{\cos[(\pi - y)p]}{\cos \pi p}$ Same as 24 except upper edge ($y = \pi$) is insulated	$\frac{1}{\pi} \frac{\sin \frac{1}{2}y \cosh \frac{1}{2}x}{\cosh x - \cos y}$ 616
26	$Y(p) = \exp(\kappa t p^2)$ Linear flow of heat in infinite solid diffusivity κ axis of x in direction of flow Cause Initial temperature Effect Temperature at time t at point x	$\frac{1}{2\sqrt{\pi\kappa t}} \exp\left(-\frac{x^2}{4\kappa t}\right)$ 710 0
27	$Y(p) = \cos(tp^2)$ Transverse oscillations of infinite elastic plate x and y axes in the plate but all points with same y coördinate have same displacement Cause Initial displacement Effect Displacement perpendicular to plate at time t of point whose coordinate is x	$\frac{1}{2\sqrt{\pi t}} \sin\left(\frac{x^2}{4t} + \frac{\pi}{4}\right)$ 752

(Continued)

Space Variable.

Cause: Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid $\times$ Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$\frac{1}{\pi}\tan^{-1}\frac{\tanh\frac{1}{2}x}{\tan\frac{1}{2}y}$ 617, 415	
$\frac{1}{\pi}\tan^{-1}\frac{\sinh\frac{1}{2}x}{\sin\frac{1}{2}y}$ 618, 415	
$\frac{1}{2}\operatorname{erf}\frac{x}{2\sqrt{\kappa t}}$ 727, 415	$\frac{1}{2}\exp(\kappa t p_0^2 + p_0 x)\operatorname{erf}\left(\frac{x}{2\sqrt{\kappa t}} + p_0\sqrt{\kappa t}\right)$ 728.1, 440
$\frac{1}{2}\left[S\left(\frac{x}{\sqrt{2\pi t}}\right) + C\left(\frac{x}{\sqrt{2\pi t}}\right)\right]$ 754, 415	$\frac{1}{4}\exp(p_0 x + itp_0^2)\operatorname{erf}\left(\frac{x}{2\sqrt{it}} + p_0\sqrt{it}\right) + \frac{1}{4}\exp(p_0 x - itp_0^2) \times \operatorname{erf}\left(\frac{x}{2\sqrt{-it}} + p_0\sqrt{-it}\right)$ 755.1, 440

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\partial\mathcal{U}Y(p)$
28	$Y(p) = \dfrac{\sin(tp^2)}{p^2}$ Same as 27, except Cause Initial velocity	$\sqrt{\dfrac{t}{\pi}}\sin\left(\dfrac{x^2}{4t}+\dfrac{\pi}{4}\right) + \dfrac{x}{2}\left[S\left(\dfrac{x}{\sqrt{2\pi t}}\right) - C\left(\dfrac{x}{\sqrt{2\pi t}}\right)\right]$ 756
29	$Y(p) = \cos(t\sqrt{1-p^2})$ Same as 19, except Cause Initial displacement multiplied by ϵ^{-t} Effect Vertical displacement multiplied by ϵ^{-t} at time t of particle whose undisturbed position is x	$\tfrac{1}{2}[\mathfrak{H}_0(x-t) + \mathfrak{H}_0(x+t)] - \dfrac{tJ_1(\sqrt{t^2-x^2})}{2\sqrt{t^2-x^2}}, \quad -t < x < t$ 619 871 2
30	$Y(p) = \dfrac{\sin(t\sqrt{1-p^2})}{\sqrt{1-p^2}}$ Same as 29 except Cause Initial velocity multiplied by ϵ^{-t}	$\tfrac{1}{2}J_0(\sqrt{t^2-x^2}), \quad -t < x < t$ 872
31	$Y(p) = \epsilon^{-\lvert yp\rvert}$ Flow of electricity in infinite thin plane x and y axes in the plane Cause Potential along x axis Effect Potential at point $(x\ y)$	$\dfrac{1}{\pi}\cdot\dfrac{\lvert y\rvert}{x^2+y^2}$ 632
32	$Y(p) = \cosh(atp)$ Transverse motion of infinite stretched elastic string, axis of x along equilibrium position of string velocity of propagation along string $= a$ Cause Initial displacement Effect Normal displacement of particle at x at time t	$\tfrac{1}{2}[\mathfrak{H}_0(x-at) + \mathfrak{H}_0(x+at)]$ 619

Cause: Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid $\times$ Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$\frac{1}{2}\left[\left(t+\frac{x^2}{2}\right)S\left(\frac{x}{\sqrt{2\pi t}}\right)+\left(t-\frac{x^2}{2}\right)\times C\left(\frac{x}{\sqrt{2\pi t}}\right)+x\sqrt{\frac{t}{\pi}}\sin\left(\frac{x^2}{4t}+\frac{\pi}{4}\right)\right]$ 757, 415	
$\frac{1}{\pi}\tan^{-1}\frac{x}{\lvert y\rvert}$ 633, 415	
$\pm\frac{1}{2}, \quad at < \pm x$ 620, 415	$\begin{cases} \pm\frac{1}{2}e^{p_0x}\cosh(atp_0), & at < \pm x \\ \frac{1}{2}e^{p_0x}\sinh(atp_0), & -at < x < at \end{cases}$ 621.4, 440

No.	Admittance $Y(p)$ Illustrative System Cause and Effect	Cause Unit Impulse Effect $\mathfrak{M}Y(p)$
33	$Y(p) = \frac{\sinh(atp)}{ap}$ Same as 32 except Cause Initial velocity	$\frac{1}{2a}, \qquad -at < x < at$ 622
34	$Y(p) = \frac{1}{\rho}\cos(\alpha\sqrt{\lvert p\rvert})e^{y\lvert p\rvert}$ Waves on deep water axis of y vertically upwards axis of x in the surface density $= \rho$, gravity constant $= g$ $y \leqq 0$ Cause Initial surface impulse along x axis Effect Velocity potential at time t at point $(x\ y)$ $h = \frac{\alpha}{\sqrt{2\pi\lvert x\rvert}}, \qquad \alpha = t\sqrt{g}$	$\frac{-y}{\pi\rho(x^2+y^2)} + \frac{i\alpha}{4\rho\sqrt{\pi}(-y+ix)^{3/2}}$ $\times \exp\left[-\frac{\alpha^2}{4(-y+ix)}\right]$ $\times \operatorname{erf}\frac{i\alpha}{2\sqrt{-y+ix}}$ $+ \frac{i\alpha}{4\rho\sqrt{\pi}(-y-ix)^{3/2}}$ $\times \exp\left[-\frac{\alpha^2}{4(-y-ix)}\right]$ $\times \operatorname{erf}\frac{i\alpha}{2\sqrt{-y-ix}}$ 845
35	$Y(p) = -\frac{\sqrt{\lvert p\rvert}}{\rho\sqrt{g}}\sin(\alpha\sqrt{\lvert p\rvert})$ Same as 34, except Effect Surface elevation at time t at point x	$\frac{\alpha}{2\pi\rho x^2\sqrt{g}} + \frac{1}{\rho\lvert x\rvert\sqrt{2\pi g\lvert x\rvert}}$ $\times \{[\cos(\frac{1}{2}\pi h^2) - \pi h^2 \sin(\frac{1}{2}\pi h^2)]C(h)$ $+ [\sin(\frac{1}{2}\pi h^2) + \pi h^2\cos(\frac{1}{2}\pi h^2)]S(h)\}$ 843
36	$Y(p) = \sqrt{g}\,\frac{\sin(\alpha\sqrt{\lvert p\rvert})}{\sqrt{\lvert p\rvert}}e^{y\lvert p\rvert}$ Same as 34, except Cause Initial surface elevation	$\frac{\sqrt{g}}{2i\sqrt{\pi(-y+ix)}}$ $\times \exp\left[-\frac{\alpha^2}{4(-y+ix)}\right]$ $\times \operatorname{erf}\frac{i\alpha}{2\sqrt{-y+ix}}$ $+ \frac{\sqrt{g}}{2i\sqrt{\pi(-y-ix)}}$ $\times \exp\left[-\frac{\alpha^2}{4(-y-ix)}\right]$ $\times \operatorname{erf}\frac{i\alpha}{2\sqrt{-y-ix}}$ 846

(Continued)

Cause: Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}[Y(p)/p]$	Cause: Unit Cisoid × Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}[Y(p)/(p-p_0)]$
$\begin{cases} \pm\frac{1}{2}t, & at < \pm x \\ \dfrac{x}{2a}, & -at < x < at \end{cases}$ 623, 415	$\begin{cases} \pm\dfrac{1}{2ap_0}e^{p_0x}\sinh(atp_0), & at < \pm x \\ \dfrac{1}{2ap_0}[e^{p_0x}\cosh(atp_0) - 1], & -at < x < at \end{cases}$ 624.2, 440
$\mp\dfrac{1}{\rho}\sqrt{\dfrac{2}{\pi g\|x\|}}[\sin(\frac{1}{2}\pi h^2)S(h) + \cos(\frac{1}{2}\pi h^2)C(h)], \quad 0 < \pm x$ 844	

No	Admittance $Y(p_1, p_2)$ Illustrative System Cause and Effect	Cause: Unit Impulse Effect: $\mathfrak{M}_1\mathfrak{M}_2 Y(p_1, p_2)$
37	$Y(p_1, p_2) = \cos [t(p_1^2 + p_2^2)]$ Transverse oscillations of infinite elastic plate, x and y axes in the plate. Cause: Initial displacement Effect: Displacement perpendicular to plate at time t of point whose coördinates are x and y.	$\frac{1}{4\pi t} \sin \frac{x^2 + y^2}{4t}$ 759, 758
38	$Y(p_1, p_2) = \exp\left(-z\sqrt{\lvert p_1^2 + p_2^2\rvert}\right)$ Velocity potential function in semi infinite incompressible fluid, x and y axes in surface of fluid, z extending down, $z \geqq 0$ Cause: Velocity potential at surface, $z = 0$. Effect: Velocity potential at point (x, y, z).	$\frac{1}{2\pi} \cdot \frac{z}{(x^2 + y^2 + z^2)^{3/2}}$ 867, 919
39	$Y(p_1, p_2) = \frac{\exp\left(-z\sqrt{\lvert p_1^2 + p_2^2\rvert}\right)}{\sqrt{\lvert p_1^2 + p_2^2\rvert}}$ Newtonian potential function in semi-infinite solid, x and y axes in face of solid, z extending into solid, $z \geqq 0$ Cause: Normal potential derivative at surface, $z = 0$. Effect: Potential at point (x, y, z).	$\frac{1}{2\pi} \cdot \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ 868, 918

(Continued)

Two Space Variables.

Cause: Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}_1\mathfrak{M}_2[Y(p_1, p_2)/(p_1 p_2)]$	Cause: Unit Cisoid $\times$ Unit Step $(-\frac{1}{2}, +\frac{1}{2})$ Effect: $\mathfrak{M}_1\mathfrak{M}_2 \dfrac{Y(p_1, p_2)}{(p_1-p_0)(p_2-p_0)}$
$\frac{1}{2}\left[S\left(\frac{x}{\sqrt{2\pi t}}\right)C\left(\frac{y}{\sqrt{2\pi t}}\right) + C\left(\frac{x}{\sqrt{2\pi t}}\right)S\left(\frac{y}{\sqrt{2\pi t}}\right)\right]$ 753; 754, 415	
$\frac{1}{2\pi}\tan^{-1}\frac{xy}{z\sqrt{x^2+y^2+z^2}}$ †	
$\frac{x}{4\pi}\log\frac{\sqrt{x^2+y^2+z^2}+y}{\sqrt{x^2+y^2+z^2}-y} + \frac{y}{4\pi}\log\frac{\sqrt{x^2+y^2+z^2}+x}{\sqrt{x^2+y^2+z^2}-x} - \frac{z}{2\pi}\tan^{-1}\frac{xy}{z\sqrt{x^2+y^2+z^2}}$ †	

† This solution was obtained by double integration of the unit impulse solution, not by the operation indicated at the head of the column. The two pairs required for this operation have not yet been found in closed form.